Diffusion approximation model for the distribution of packet travel time at sensor networks

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***Abstract*— We propose a model based on diffusion approximation to estimate the probability density function of the distribution of a packet travel time in a multihop wireless sensor network. In its general form, the model assumes that the propagation medium and the distribution of relay nodes may be heterogeneous in space and that the system characteristics may change over time. It consideres also the retransmission in case of a packet loss.**

Keywords: *diffusion approximation, transient analysis, wireless networks, sensor networks*

I. Introduction

Prediction of a packet travel time in wireless sensor networks is still an open issue. The sensor networks, see e.g. [1] consist of a large number of simple nodes scattered randomly over a certain area, having ability to route packets to their neighbours and finally to the sink which collects the data sent to it via multihop transmission.

The topology of such networks is in most of cases uncertain and it changes in time (due to nodes movement or failures), hence special routing algorithms were proposed, e.g. [5], [6] to face this situation. As it is also hard to introduce global addressing, the routing decision must be made without the complete information about the network. We consider the same network model as in [4] – a packet wireless network in which nodes are distributed over an area, but where we do not know about the presence, exact location, or reliability of nodes. The packets are forwarded to a node witch is most probably nearer to the destination, but it is also possible that a transmission may actually move the packet further away from the sink or send it to a node which is in the same distance to the destination (see for example [8]). It may also happen that a packet cannot be forwarded any further, that the intermediate node has a failure, or that the packet is lost through noise or some other transient effect. In that case, the packet may be retransmitted after some time-out period has elapsed, either by the source or from some intermediate storage location on the path which it traversed before it was lost.

II. Model formulation

Recently Gelenbe [4] proposed a model based on diffusion approximation to estimate the mean transmission time from a source to destination in a random multihop medium. In this model a value of the diffusion process represents the distance defined as the number of hops between the transmitted packet and its destination (sink). Due to complex topology and transmission constraints, it is not sure that each one-hop transmission makes this distance shorter and the changes of the distance may be considered as random process. This justifies the use of diffusion process to characterise it. Diffusion approximation is a classical method used in queueing theory to represent a queue length or queueing time e.g. [3], in case of general independent distributions of interrarival and service times. Diffusion process is a continuous stochastic process but it is used to approximate some discrete processes, see [2], like – as mentioned above – the number of customers in a queue; here it represents the number of hops remaining to packet to the destination.

If $N(t)$ denotes the number of hops remaining to destination at time t, we construct a diffusion process $X(t)$ such that its density function $f(x,t;x_0)$ approximates probability distribution $p(n,t;n_0)$ of the process $N(t)$, $N(0) = n_0$: $f(n,t;n_0) \approx p(n,t;n_0)$. The density function $f(x,t;x_0)$

$$f(x,t;x_0)dx = P[x \le X(t) < x + dx \mid X(0) = x_0]$$

is defined by the diffusion equation

$$\frac{\partial f(x,t;x_0)}{\partial t} = \frac{\alpha}{2}\frac{\partial^2 f(x,t;x_0)}{\partial x^2} - \beta\frac{\partial f(x,t;x_0)}{\partial x}\ , \qquad (1)$$

where the parameters β and α define respectively the mean and variance of infinitesimal changes of the diffusion process. To maintain them similar to the considered process $N(t)$, they should be chosen as

$$\begin{aligned} \beta &= \lim_{\Delta t \to 0} \frac{E[N(t+\Delta t) - N(t)]}{\Delta t} \\ \alpha &= \lim_{\Delta t \to 0} \frac{E[(N(t+\Delta t) - N(t))^2] - (E[N(t+\Delta t) - N(t)])^2}{\Delta t} \end{aligned}$$

In general, the parameters may depend on time and on the current value of the process, $\beta = \beta(x,t)$ and $\alpha = \alpha(x,t)$, as the propagation medium and distribution of relay nodes may be heterogeneous in space and the system characteristics

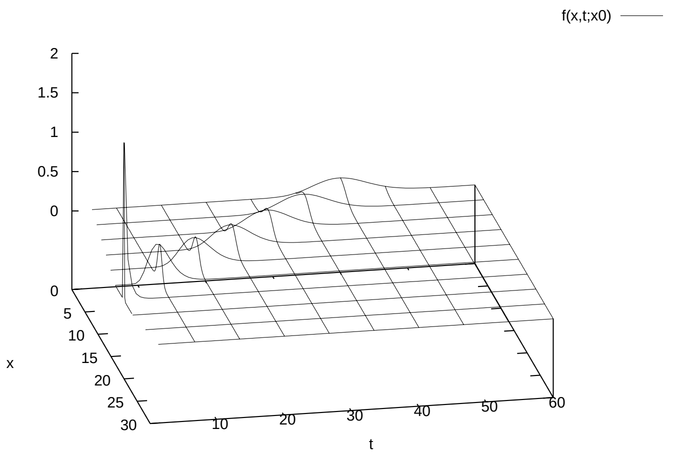

Fig. 1. Density function $\phi(x,t;x_0)$ of the diffusion process with absorbing barrier, $x_0 = 20$, $\alpha = 0.1$, $\beta = -0.5$.

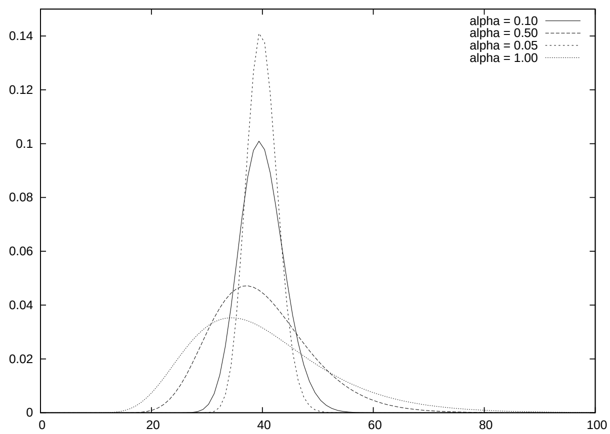

Fig. 2. Distribution of first passage time from x_0 to 0, $\gamma_{x_0,0}(t)$, $x_0 = 20$, $\beta = -0.5$, $\alpha = 0.05, 0.1, 0.5, 1.0$.

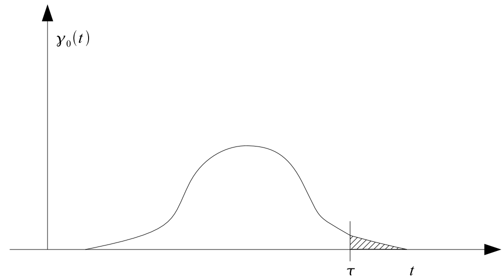

Fig. 3. Probability $p_T = \int_T^\infty \gamma_{x_0,0}(t)dt$ that a packet at the moment T is still on its way.

may change over time. We include this case in the proposed approach.

Gelenbe in [4] constructs an ergodic process going repetitively from starting point to zero and considers its steady-state properties. Here, to obtain the distribution (and not only the mean transmission time as given in [4]), we use transient solution of diffusion equation and we consider only a single process. Let us repeat that the process starts at $x_0 = N$ and ends when it successfully comes to the absorbing barrier at $x = 0$; the position x of the process corresponds to the current distance between the packet and its destination, counted in hops.

III. Model without deadline and without losses

In this simplest case we consider diffusion equation (1) with constant coefficients, supplemented with absorbing barrier at $x = 0$. This barrier is expressed by the boundary condition $\lim_{x\to 0} f(x,t;x_0) = 0$. The process starts at x_0: $X(0) = x_0$ and ends when it comes to the barrier. The diffusion process is defined at the interval $(0,\infty)$. Let us denote the solution of the diffusion equation in this case by $\phi(x,t;x_0)$; it is obtained using mirror method, see e.g. [2]

$$\phi(x,t;x_0) = \frac{1}{\sqrt{2\Pi\alpha t}}\left[e^{-\frac{(x_0-x-|\beta|t)^2}{2\alpha t}} - e^{\frac{2\beta x_0}{\alpha}} e^{-\frac{(2x_0-|\beta|t)^2}{2\alpha t}}\right] .$$

Fig. 1 presents a plot of the function $\phi(x,t;x_0)$. The function allows us to determine the first passage time from $x = x_0$ to $x = 0$ and to estimate this way the density of a packet transmission time through x_0 hops from a node to the sink:

$$\begin{aligned}\gamma_{x_0,0}(t) &= \lim_{x\to 0}[\frac{\alpha}{2}\frac{\partial}{\partial x}\phi(x,t;x_0) - \beta\phi(x,t;x_0)] \\ &= \frac{x_0}{\sqrt{2\Pi\alpha t^3}} e^{-\frac{(x_0-|\beta|t)^2}{2\alpha t}} .\end{aligned}$$

Some exemplary curves of $\gamma_{x_0,0}(t)$ are presented in Fig. 2.

IV. Introduction of the deadline

Denote by T the time after which a packet is considered lost and is retransmitted by the source. Knowing the density $\gamma_{x_0,0}(t)$ of the travel time from x_0 to 0, we can determine the probability $p_T = \int_T^\infty \gamma_{x_0,0}(t)dt$ that a packet at the moment T is still on its way – see Fig. 3.

In the model, at $t = T$ we shift this probability mass p_T to x_0 and we restart the diffusion process. We may of course introduce an additional delay before the restart.

V. Modelling heterogeneous medium and losses

To reflect the fact that the transmission conditions may be different for each hop, the diffusion interval is divided into unitary intervals corresponding to single hops. The subintervals are separated by fictive barriers allowing us to balance the probability density flows between them. We limit the whole interval to a value corresponding to the size of the network $x \in [0, D]$, the starting point x_0 is somewhere inside this interval. As in general $\beta < 0$ (i.e. a packet has a tendency of going towards the sink), thr probability of reaching the right barrier by the diffusion process is small. If however the process reaches the right barrier, it is immediately sent to the point $x = D - \varepsilon$ and the process is continued.

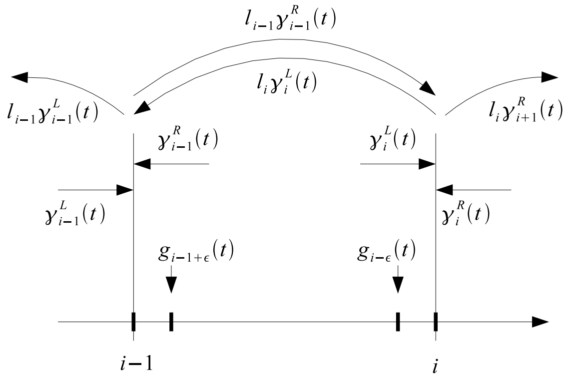

Fig. 4. Diagram of probability mass circulation due to nonhomegonous diffusion parameters and due to losses with probability l_i at i-th hop, $i = 2, \dots, D-1$.

An interval i, $x \in [i-1, i]$ represents the packet transmission when it is i hops distant from the sink. We assume that parameters β_i, α_i are proper to this interval and we assume also the loss probability l_i within this interval.

When the process approaches one of these barriers, for example the barrier i, it acts as an absorbing one, but then immediately the process reappears at the other side of the barrier with probability $(1 - l_i)$ (probability of successful transmission) or with probability l_i it comes to the node that it visited previously, i.e. to the barrier at $x = i+1$ or at $x = i-1$.

Let $\gamma_i^L(t)$ represent the flow coming to the barrier placed at $x = i$ from its left side and $\gamma_i^R(t)$ be the flow coming to this barrier from its right side. The flows start diffusion processes at both sides of the barrier, respectively $\gamma_i^R(t)$ reappears at $x = i - \varepsilon$ and $\gamma_i^L(t)$ at $x = i + \varepsilon$ but the intensities of the trespassing flows are reduced by flows corresponding to the loss of packet during the previous hop transmission. Thus inside the interval i the process starts with intensities

$$\begin{aligned} g_{i-1+\varepsilon}(t) &= (1-l_i)\gamma_i^L(t) + l_{i-1}\gamma_{i-1}^R(t) \\ g_{i-\varepsilon}(t) &= (1-l_{i-1})\gamma_{i-1}^R(t) + l_i\gamma_i^L(t) \end{aligned}$$

where $g_{i-1+\varepsilon}(t)$ and $g_{i-\varepsilon}(t)$ are the probability densities that the diffusion process starts at time t at the point $x = i-1+\varepsilon$ and $x = i - \varepsilon$.

If we assume that the loss may be repaired by sending the lost packet from the neighbouring node, the flow $\gamma_i^L(t)l_i$ is sent to $x = i-1+\varepsilon$ and the flow $\gamma_i^R(t)l_i$ is sent to $x = i+1-\varepsilon$.

The circulation of probability mass for i-th interval, representing i-th hop, is presented in Fig. 4.

If the lost packets are retransmitted a certain delay, e.g. after a random time distributed with density function $l(t)$, [if this time is constant and equal r then $l(t) = \delta(t-r)$], we rewrite the above equations as

$$\begin{aligned} g_{i-1+\varepsilon}(t) &= (1-l_i)\gamma_i^L(t) + l_{i-1}\gamma_{i-1}^R(t) * l(t) \\ g_{i-\varepsilon}(t) &= (1-l_{i-1})\gamma_{i-1}^R(t) + l_i\gamma_i^L(t) * l(t) \end{aligned}$$

where $*$ denotes the operation of convolution.

Within each subinterval we have diffusion process with two absorbing barriers, e.g. for i-th interval at $x = i-1$ and $x = i$ and with two points when the process is started, at $i - 1 + \varepsilon$ with intensity $g_{i-1+\varepsilon}(t)$ and at $i - \varepsilon$ with intensity $g_{i-\varepsilon}(t)$.

The density of the diffusion process started at x_0 within an interval $(0, N)$ having the absorbing barriers at $x = 0$ and $x = N$ has the form, see [2]

$$\phi(x,t;x_0) = \begin{cases} \delta(x-x_0), t = 0 \\ \dfrac{1}{\sqrt{2\Pi\alpha t}} \displaystyle\sum_{n=-\infty}^{\infty} \Big\{ \exp\left[\frac{\beta x_n'}{\alpha} - \frac{(x - x_0 - x_n' - \beta t)^2}{2\alpha t} \right] \\ - \exp\left[\frac{\beta x_n''}{\alpha} - \frac{(x-x_0-x_n''-\beta t)^2}{2\alpha t} \right] \Big\}, t > 0 \,, \end{cases}$$

where $x_n' = 2nN$, $x_n'' = -2x_0 - x_n'$.

The density $f_i(x,t;\psi)$ may be expressed as a superposition of functions $\phi_i(x,t;x_0)$ at the interval $(i-1, i)$

$$\begin{aligned} f_i(x,t;\psi_i) &= \phi(x,t;\psi_i) + \int_0^t g_{i-1+\varepsilon}(\tau)\phi(x, t-\tau; i-1+\varepsilon)d\tau \\ &+ \int_0^t g_{i-\varepsilon}(\tau)\phi(x, t-\tau; i-\varepsilon)d\tau \,. \end{aligned}$$

where the function ψ_i represents the initial conditions.

The flows $\gamma_{i-1}^L(t)$ and $\gamma_i^R(t)$ for the i-th interval are obtained as

$$\gamma_{i-1}^R(t) = \lim_{x \to (i-1)} [\frac{\alpha_i}{2} \frac{\partial f_i(x,t;\psi_i)}{\partial x} - \beta_i f_i(x,t;\psi_i)]$$

$$\gamma_i^L(t) = -\lim_{x \to (i)} [\frac{\alpha_i}{2} \frac{\partial f_i(x,t;\psi_i)}{\partial x} - \beta_i f_i(x,t;\psi_i)].$$

It is much easier to solve the system of all the above equations when they are inverted with the use of Laplace transform: all convolutions become in this case products of transforms, the Laplace transform of the function $\phi(x,t;x_0)$ is

$$\begin{aligned} \bar{\phi}(x,s;x_0) &= \frac{\exp[\frac{\beta(x-x_0)}{\alpha}]}{A(s)} \sum_{n=-\infty}^{\infty} \Big\{ \exp\left[-\frac{|x - x_0 - x_n'|}{\alpha} A(s) \right] \\ &- \exp\left[-\frac{|x - x_0 - x_n''|}{\alpha} A(s) \right] \Big\} \,, \end{aligned}$$

where $A(s) = \sqrt{\beta^2 + 2\alpha s}$.

The final solution $f_i(x,t;\psi_i)$ is obtained by numerical inversion of its Laplace transform $\bar{f}_i(x,s;\psi_i)$. In examples below we used Stehfest algorithm [7].

Fig. 6 shows the density $f(x,t;x_0)$ of the remaining distance to complete the transfer calculated for time moments $t = 10, 20$ and 30 if $\beta = -0.4$ and $\alpha = 0.54$. The packet transmission started at the distance $x_0 = 10$ from its destination.

Figure 7 displays the density $\gamma_{x_0,0}(t)$ of the first passage time from x_0 to $x = 0$, hence the approximation of the transmission time density. The values of chosen parameters are:

$$\begin{aligned} &\beta = -0.4, \;\; \alpha = 0.54 \quad \longrightarrow \quad (\pi_{-}1, \pi_0, \pi_{+}1) = (0.55, 0.30, 0.15), \\ &\beta = -0.2, \;\; \alpha = 0.54 \quad \longrightarrow \quad (\pi_{-}1, \pi_0, \pi_{+}1) = (0.40, 0.40, 0, 20), \\ &\beta = -0.3, \;\; \alpha = 0.81 \quad \longrightarrow \quad (\pi_{-}1, \pi_0, \pi_{+}1) = (0.60, 0.10, 0.30). \end{aligned}$$

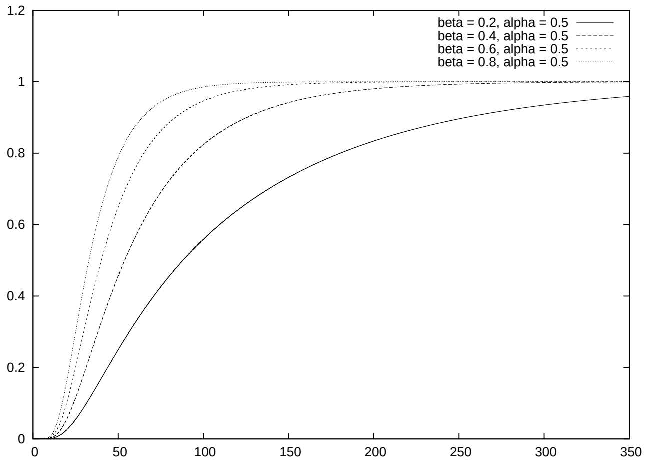

Fig. 5. Probability $p(0, t)$ if the starting point is $x_0 = 10$, diffusion interval $x \in [0, 20]$, $\alpha = 0.5$, and β is variable ($\beta = 0.2, 0.4, 0.6, 0.8$).

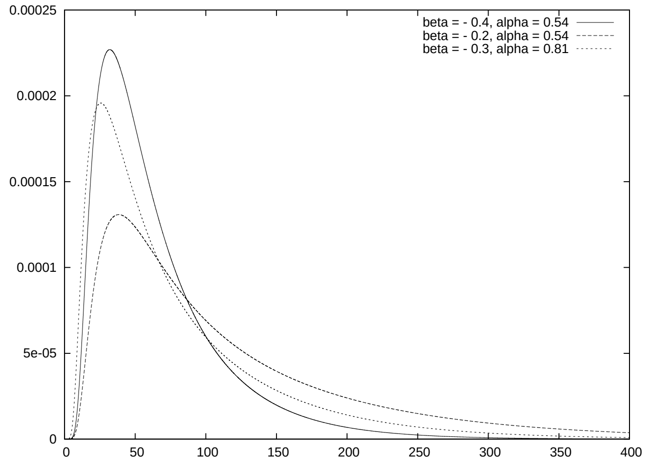

Fig. 7. The density function $\gamma_{x_0,0}(t)$ of the first passage time from x_0 to $x = 0$, i.e. the approximation of the transmission time density.

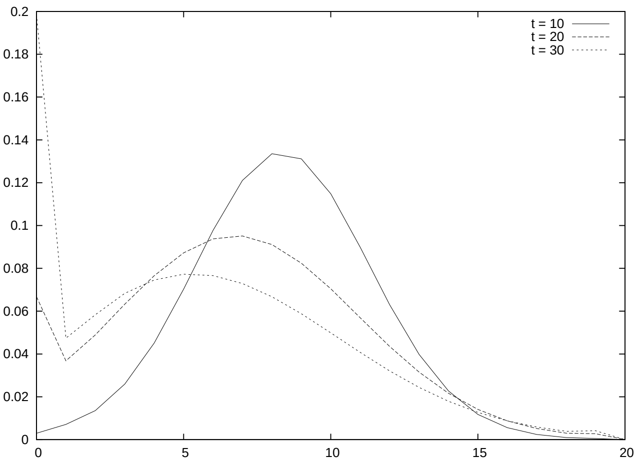

Fig. 6. The density function $f(x, t; x_0)$ of the distance to destination at time $t = 10, 20$ and 30. The network has parameters $\beta = -0.4$ and $\alpha = 0.54$ and packet initial position is $x_0 = 10$.

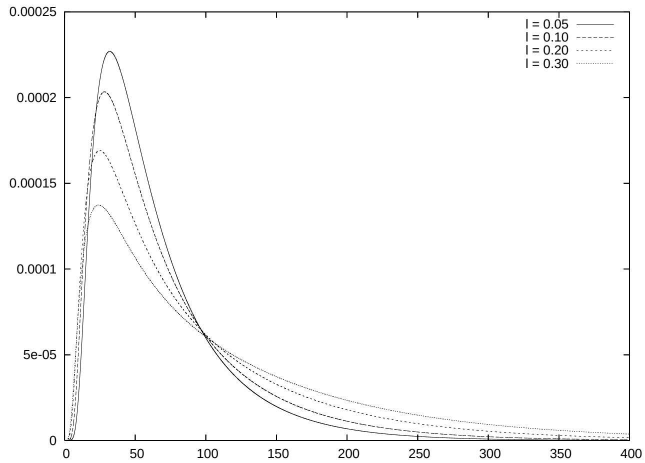

Fig. 8. The impact of loss ratio l ($l = 0.05, 0.1, 0.2, 0.3$) on the density function of transfer time at a network with $\beta = -0.4$ and $\alpha = 0.54$.

Figure 8 presents the influence of the loss rate l ($l = 0.05, 0.1, 0.2, 0.3$) on the density of transmission time for parameters $\beta = -0.4$ et $\alpha = 0.54$.

VI. Conclusions

Owing to the introduction of the transient state analysis, the presented model seems to capture more parameters (time-dependent and heterogeneous transmission, the presence of specific to each hop losses) of a sensor network transmission time than the existing models, also based on diffusion approximation. It gives also more detailed results: the density function of a packet travel time instead of its mean value. Numerical results prove that the model is operational.

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