Spatial Receive Diversity versus Beamforming: On The Optimal Antenna Usage Strategies with Multiantenna RAKE Multiuser Receivers over Multipath Fading CDMA Channels

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Abstract

In this paper, we analyze the optimal antenna usage strategies with multiantenna RAKE multiuser receivers over time-varying multipath Rayleigh/Ricean fading BPSK-CDMA channels to elucidate the optimal partitioning of the receive antennas either for spatial diversity combining or for interference nulling via maximum-SIR beamforming that are competing strategies to enhance detection performance. We propose a novel hybrid spatial diversity-combining/beamforming space-time RAKE receiver structure and relying on random spreading analysis for generality, we derive and analyze the average bit error rate of the receiver as a function of both the antenna partioning between diversity reception and beamforming as well as other key system parameters such as the load factor, the order of frequency diversity resolvable and average received bit SNR per path.

Keywords: code-division multiple-access, RAKE receiver, beamforming, antenna-array processing, multipath fading, diversity techniques.

1. MOTIVATION AND OUTLINE OF THE PAPER

In this paper, the problem of finding the optimal partioning of receive antennas between spatial diversity reception and beamforming that are competing strategies to enhance multiuser detection performance is adressed. We propose a novel hybrid diversity-combining/beamforming multiantenna space-time RAKE receiver structure and analyze its average bit error rate and average multiuser power efficiency over time-varying multipath Rayleigh/Ricean channels to dechipher the optimal partioning problem. Linear/nonlinear multiantenna multiuser detection for both uncoded/coded transmission for CDMA systems are previously attacked in the literature in [1-3]. However, in any of these significant contributions and many other similar references therein as well, such an optimal antenna partioning problem to elucidate the trade-off between antenna diversity reception and beamforming is not addressed to our knowledge, thus, leaving the problem as an open research issue.

The paper is organized as follows. The system model and the structure of the linear RAKE multiuser receiver with hybrid diversity-beamforming reception strategy are presented in Section II. In Section III, the average bit error probability of the receiver are derived over Rayleigh/Ricean time-varying multipath fading channels. Numerical results and discussions are presented in Section IV, and conclusions are drawn in Section V.

2. SYSTEM MODEL and HYBRID RECEPTION SPACE-TIME RAKE RECEIVER

We consider a chip/symbol synchronous frequency-selective multipath Rayleigh/Ricean fading channel of W_s Hz system bandwidth for an uplink CDMA communication system of K users randomly located in a single cell with a single transmit antenna and a base station uniform linear antenna array of M sensor elements. Transmitted signal from the kth user is given by:

$$s_k(t) = A_k b_k c_k(t), \tag{1}$$

where $A_k = \sqrt{E_b^k}$ is the amplitude of the kth user under unity AWGN power spectral density assumption; i.e. $N_0 = 1$. E_b^k is the energy per bit of the kth user, $b_k \in \{-1, 1\}$ is the transmitted antipodal coherent BPSK symbol of the kth user i.i.d. with equal probability and independent over time and users, and $c_k(t)$ represents the spreading waveform of the kth user, which is given by:

$$c_k(t) = \sum_{l=0}^{L-1} c_k(l) v(t - lT_c) \tag{2}$$

where $\mathbf{c}_k = [c_k(1)\ c_k(2)...c_k(L)]^T$ is the spreading code vector of kth user, $v(t)$ is the unit-energy Nyquist chip pulse shaping filter, $T_c \simeq \frac{1}{W_s}$ is the chip interval, $T_b \simeq \frac{1}{R_b}$ is the bit interval, R_b is the data rate in bits/sec and the spreading factor L is defined as $L = \frac{T_b}{T_c}$. Furthermore, the number of propagation paths in the multipath channel that is equivalent to the order of frequency diversity resolvable for all users is given by

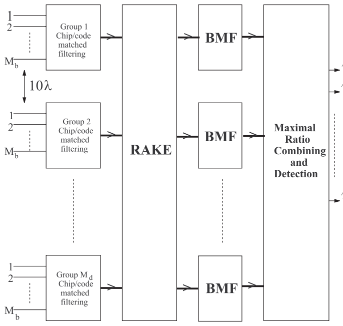

Figure 1: Block diagram of the hybrid reception space-time RAKE receiver.

$D = \lceil T_m W \rceil \simeq \frac{T_m}{T_c} = \frac{W}{(\Delta f)_c}$, where T_m is the delay spread of the multipath channel and $(\Delta f)_c = \frac{1}{T_m}$ is the coherence bandwidth of the channel.

The receiver block diagram we propose for linear space-time RAKE receiver with hybrid reception strategy is depicted in Fig. 1 where M available antennas are divided into M_d spatial diversity groups of M_b beamforming antennas for spatial interference supression filtering such that $M = M_d \times M_b$. The diversity groups are assumed to be spatially seperated by at least $10\lambda_c$ to assure independence of the fading between diversity groups, while the beamforming antennas in each diversity group are seperated with a distance $d_m \leq \frac{\lambda_c}{2}$, where $\lambda_c = \frac{c}{f_c}$ is the carrier wavelength for carrier frequency f_c and $c = 3 \times 10^8$ m/s is the speed of light. Thus, assuming that the phase of the signals at the first antenna element in each beamforming group is zero, the phase lead of each signal at ith antenna element of the same group is given by $\frac{2\pi}{\lambda_c} i d_m \sin(\theta)$ where θ is the DOA of the signal in radians.

In maximum-SIR beamforming [4], the antenna array spatial response as a function of DOA θ is given by:

$$F(\theta) = \sum_{i=0}^{M-1} v_i \exp\left(j \frac{2\pi}{\lambda_c} i d_m \sin(\theta)\right) = \mathbf{v}^T \mathbf{a}(\theta), \quad (3)$$

where $\mathbf{v}$ is the vector of beamforming coefficients of the form $v_i = C_i \exp(j i \alpha)$ and $\mathbf{a}(\theta) = [1 \exp(j\theta) \exp(j2\theta) ... \exp(j(M-1)\theta)]^T$ is the array steering vector associated with DOA θ. Thus, if the parameters α and C_i's are set to $\alpha = -\frac{2\pi}{\lambda_c} d_m \sin(\theta_0)$ and $C_i = \frac{1}{M}$ $\forall i$, the 0-dB maximum normalized array response occurs at DOA θ_0; i.e. the array is steered towards θ_0 and all other interfering signals arriving from different directions will be supressed depending on the shape of the array response.

Due to the large spacing between spatial diversity groups, we assume that the fading process of each path of each user is independent over spatial diversity groups, and thus, the fading process for each path of each user at antennas within each beamforming group is just the phase-shifted versions of the base one at the first antenna of the group with zero phase lead. Hence, there exist $M_d \times K \times D$ independent fading processes in the system. Furthermore, for the distribution on the complex channel coefficients, we focus on complex-Gaussian fading processes for the diversity channels of each user that are jointly stationary, ergodic and mutually-independent with a uniform multipath intensity profile and unity time-average power. Thus, this results in circularly-symmetric, mutually-independent and ergodic complex-Gaussian channel coefficients with normalized unity mean-squared values: $\mathrm{E}\{|h_{kdm}|^2\} = 1, \forall k \in \{1, ..., K\}, d \in \{1, ..., D\}, m \in \{1, ..., M_d\}$, hence modelling a Rayleigh fading or Ricean fading multipath environment depending on the zero-mean and nonzero-mean behaviour of the fading processes respectively. The $K \times D$ paths of all users have their associated distinct DOAs with respect to the linear antenna array normal, i.e. θ_{kd}, $\forall k \in \{1, ..., K\}, d \in \{1, ..., D\}$, that are assumed to be perfectly known at the receiver, uniformly and independently distributed between $[-\pi, \pi]$, and constant over the interval of observation such that no adaptivity in tracking them are required. The time variation of the paths of users due to the mobility are also further defined by their individual inverse Doppler spread, i.e. $(\Delta t)_{kd} = \frac{1}{B_{kd}}$, where the Doppler spread of the kdth channel for kth user's mobile speed V_k is given by $B_{kd} = \frac{V_k}{\lambda_c} \cos(\theta_{kd})$. Moreover, the transmitters are assumed to have no knowledge on the channel state information while the receiver is able to perfectly track the channel state information for combining purposes.

After chip-matched filtering via the perfect knowledge of the spreading codes of all users and their relative delays assumed to be integer multiples of the chip period and distributed in ascending order between $[0, T_m)$ such that no interchip/intersymbol interference occurs for the sufficiency of the one-shot detection, the received vectorial discrete-time signal in an arbitrary bit signalling interval can be written in compact vector-matrix form as:

$$\mathbf{y} = \mathbf{RAHCb} + \mathbf{w}, \mathbf{y} \in C^{MKD \times 1} \quad (4)$$

where $\mathbf{R}$ is the $MKD \times MKD$ crosscorrelation matrix, $\mathbf{A} \in C^{MKD \times M_d KD}$ is the block-diagonal matrix of array steering vectors; i.e. $\mathbf{A} = \mathrm{diag}(\mathbf{A}_1, \mathbf{A}_2, ..., \mathbf{A}_k)$, where $\mathbf{A}_k = \mathrm{diag}(\mathbf{a}_k(\theta_1), \mathbf{a}_k(\theta_2), ..., \mathbf{a}_k(\theta_{M_d D})) \in C^{MD \times M_d D}$, $\mathbf{H} \in C^{M_d KD \times K}$ is the block-diagonal channel coefficients matrix; i.e. $\mathbf{H} = \mathrm{diag}(\mathbf{h}_1, \mathbf{h}_2, ..., \mathbf{h}_K)$, where $\mathbf{h}_k = [h_1, h_2, ..., h_{M_d D}]^T$, $\mathbf{C} \in R^{K \times K}$ is the diagonal amplitudes matrix; i.e $C = diag\left(\sqrt{E_b^1}, \sqrt{E_b^2}, ..., \sqrt{E_b^K}\right)$, $\mathbf{b} \in \{-1, +1\}^K$ is the K-dimensional vector of i.i.d. equally-likely antipodal coherent BPSK symbols of all users and $\mathbf{w} \in C^{MKD \times 1}$ is the circulary-symmetric zero-mean complex-Gaussian noise vector with crosscovariance matrix $\mathrm{E}\{\mathbf{w}\mathbf{w}^H\} = N_0\mathbf{R}$, where the one-sided noise power spectral density N_0 is assumed to be unity without loss of generality.

Following the front-end chip-matched filtering, no interference suppression filtering is applied corresponding to the RAKE multiuser detection with linear estimator matrix $\mathbf{M}^{(\mathrm{RAKE})} = \mathbf{I}_{MKD\times MKD}$. Following multiuser RAKE reception, spatial beamforming is applied for each path of each user at all diversity groups via the assumed perfect knowledge of the DOAs, and the paths of each user at different diversity groups are then combined by maximal-ratio combining via the assumed perfect knowledge of the channel coefficients to form the final decision statistics. Finally, the bits of each user are detected by taking the sign of the final decision statistic due to antipodal BPSK modulation.

3. AVERAGE BIT ERROR PROBABILITY

In this section, we derive the average bit error probability and average multiuser power efficiency of the hybrid reception strategy space-time RAKE receiver presented in Section II over time-varying Rayleigh/Ricean fading channels. For the analysis, we employ spherical Gaussian random spreading where the BPSK symbols at the transmitters are assumed to be spread with complex-Gaussian random spreading sequences of processing gain L, $c_k = (1/\sqrt{L})\,[c_{k1}\ \ c_{k2}...c_{kL}]$, where each chip corresponds to a circularly-symmetric complex-Gaussian, zero-mean, unity-variance random variable that are independent over chip index and users.

Due to the complex-Gaussian random spreading sequences where the real and imaginary parts of each chip is independently and identically distributed with zero mean and variance $\frac{1}{2}$, the diagonal elements of $\mathbf{R}$ are distributed according to the central chi-squared distribution $\chi^2_{2L}(1, \frac{1}{L})$ of $2L$ degrees of freedom with unity mean and variance $\frac{1}{L}$ due to the scaling by $\frac{1}{\sqrt{L}}$. Furthermore, when the chip-synchronism is perfect and the path delays are integer multiples of the chip period T_c, the off-diagonal elements of $\mathbf{R}$ converge by De Moivre-Laplace Central Limit Theorem to the circularly-symmetric complex-Gaussian distribution $N_c(0, \frac{1}{L})$ with zero mean and variance $\frac{1}{L}$. In such modelling for the random crosscorrelation matrix, the convergence $\mathbf{R} \rightarrow \mathbf{I}$ in the wideband regime as $L \rightarrow \infty$ accurately verifies the averaging out of MAI within systems where long spreading sequences are employed such as in IS-95.

Without loss of generality, let us focus on the detection of the first user with average bit SNR per path $\mathrm{SNR_b} = E_b^1$ due to unity path power/unity noise power spectral density assumptions and set the rest $K-1$ interferers' relative power with respect to the first user by the near-far ratio (NFR)

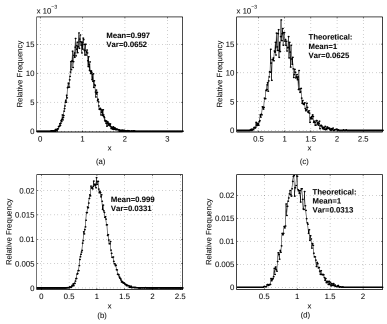

Figure 2: (a): Average histogram of eigenvalues of diagonal submatrices at D=2, L=16. (b): Histogram of a chi-squared distributed rv with individual component variances 1/2 scaled by 1/L=1/16. (c): Average histogram of eigenvalues of diagonal submatrices at D=3, L=32. (d): Histogram of a chi-squared distributed rv with individual component variances 1/2 scaled by 1/L=1/32.

as, NFR$=\frac{\mathrm{SNR}_\mathrm{b}^k}{\mathrm{SNR_b}}$, where $\mathrm{SNR}_\mathrm{b}^k = E_b^k$ is equal for all interfering users. Then, the soft-output random SIR of the final decision statistic of kth user with hybrid reception space-time linear RAKE multiuser receiver can be written compactly as:

$$\mathrm{SIR}_k^{(\mathrm{RAKE})} = \frac{\mathbf{h}_k^H\mathbf{R}_{kk}^H\mathbf{\Phi}_{kk}\mathbf{R}_{kk}\mathbf{h}_k\mathrm{SNR_b}}{\sum_{\acute{k}=2}^{K}\mathbf{h}_{\acute{k}}^H\mathbf{R}_{k\acute{k}}^H\mathbf{\Phi}_{\acute{k}\acute{k}}\mathbf{R}_{k\acute{k}}\mathbf{h}_{\acute{k}}.\mathrm{NFR}.\mathrm{SNR_b}+|n_k^{(\mathrm{RAKE})}|^2}, \qquad (5)$$

where $\mathbf{R}_{kk}$ and $\mathbf{R}_{k\acute{k}}$ are the $M_dD \times M_dD$ kkth diagonal and $M_dD \times M_dD$ $k\acute{k}$th off-diagonal random submatrices of the Hermitian crosscorrelation matrix $\mathbf{R}$ formed by the crosscorrelation values between paths of all users at first antenna elements of each diversity group and the matrices $\mathbf{\Phi}_{ij} = \mathbf{A}_i^H\mathbf{A}_j$ is the diagonal matrix of the beamforming power loss random variables defined as $\mathbf{\Phi}_{ij}(\theta_n, \theta_m) = \left|\mathbf{a}_i^H(\theta_n)\mathbf{a}_j(\theta_m)\right|^2$ that is equivalent to unity if $i = j$ due to normalized beamforming coefficients. Furthermore, the noise component $|\ n_k^{(\mathrm{RAKE})}\ |^2$ can be expanded as $|\ n_k^{(\mathrm{RAKE})}\ |^2 = \mathbf{h}_k^H\mathbf{A}_k^H\mathbf{w}_k\mathbf{w}_k^H\mathbf{A}_k\mathbf{h}_k$.

Most of the results in the random-matrix theory literature concentrate on certain symmetric matrices such as the Wishart/Wigner ensembles that are *unitarily-distribution-invariant* (UDI) orthogonal/unitary random matrix ensembles. However, the diagonal random submatrices $\mathbf{R}_{kk}$ of $\mathbf{R}$ are not UDI and there currently exists no presented mathematical solution in the literature within our knowledge for the joint eigenvalue distribution or the average density of the eigenvalues for such random matrices. Nonetheless, we show by Monte-Carlo simulations in Fig. 2 that the average eigenvalue density f(λ) of the diagonal submatrices $\mathbf{R}_{kk}$ of $\mathbf{R}$ is χ^2 distributed with $2L$ degrees of freedom scaled by $\frac{1}{L}$ having unity mean and variance $\frac{1}{L}$ especially when L is sufficiently large.

On the other hand, for the off-diagonal matrices $\mathbf{R}_{k\acute{k}}$, by real-complex partitioning, $\mathbf{R}_{k\acute{k}} = \mathbf{A} + \text{sqrt}(-1)\mathbf{B}$, where $\mathbf{B}$ is a diagonal matrix of i.i.d. $N(0, \frac{1}{2L})$ random variables and $\mathbf{A}$, if scaled by $\sqrt{2}\mathbf{I}$, is a $D \times D$ proper Hermitian complex-Gaussian random matrix with common variance $\frac{1}{L}$. The magnitude-ordered real eigenvalues of a proper $n \times n$ Hermitian complex-Gaussian random matrix with common variance of σ^2 has *Wigner density* [5], and the corresponding average eigenvalue density that is zero mean is given by , $p(\lambda) = \frac{1}{n\sigma\sqrt{2}} \sum_{k=0}^{n-1} \left[\varphi_k(\frac{\lambda}{\sigma\sqrt{2}})\right]^2$, having sum of mean-squared values derived as $E\{\lambda^2\} = n\sigma^2$ with $\varphi_k(x) = \frac{1}{(2^k k! \sqrt{\pi})^{\frac{1}{2}}} H_k(x) \exp(-\frac{x^2}{2})$ being the *Hermite-functions* in terms of the orthogonal *Hermite-polynomials* $H_k(x) = (-1)^k \exp(x^2).(\frac{d^k}{dx^k} \exp(-x^2))$.

A major methodology for performance analysis in interference-limited CDMA systems is to assume the Gaussianity of the total MAI plus noise terms and to use the total mean-squared value of the MAI plus background noise terms as the effective noise level. The Gaussianity of the total MAI plus noise terms in the output soft decision statistics of various multiuser receivers are validated both theoretically and experimentally in the literature [6]. In this case, the SIR definition in (5) turns into:

$$\text{SIR}_k^{(\text{RAKE})} = \frac{\mathbf{h}_k^H \mathbf{R}_{kk}^H \mathbf{\Phi}_{kk} \mathbf{R}_{kk} \mathbf{h}_k \text{SNR}_\text{b}}{\text{E}\left\{\sum_{\acute{k}=2}^{K} \mathbf{h}_{\acute{k}}^H \mathbf{R}_{k\acute{k}}^H \mathbf{\Phi}_{k\acute{k}} \mathbf{R}_{k\acute{k}} \mathbf{h}_{\acute{k}} .\text{NFR}.\text{SNR}_\text{b} + |n_k^{(\text{RAKE})}|^2\right\}}, \tag{6}$$

which can be written by exploiting the distribution-invariance of circularly-symmetric zero-mean Gaussian random vectors upon linear unitary transforms via eigenvalue decomposition of $\mathbf{R}_{kk}$ and $\mathbf{R}_{k\acute{k}}$ matrices as:

$$\text{SIR}_k^{(\text{RAKE})} = \frac{\sum_{d=1}^{M_d D} \text{E}\{\lambda_{kkd}^2\} \text{E}\{h_{kd}^2\} \text{SNR}_\text{b}}{\sum_{\acute{k}=2}^{K} \sum_{d=1}^{M_d D} \text{E}\left\{\Phi_{k\acute{k}d}\right\} \text{E}\left\{\left|\lambda_{k\acute{k}d}\right|^2\right\} \text{E}\left\{\left|h_{\acute{k}d}\right|^2\right\} .\text{NFR}.\text{SNR}_\text{b} + \text{E} \mid \left\{n_k^{(\text{RAKE})} \mid^2\right\}}, \tag{7}$$

that is also valid under Ricean fading due to the quadratic nature of the rvs in the definition of SIR.

For the derivations in our analysis, to average performance metrics with respect to the average eigenvalue density of the crosscorrelation matrices and the channel distributions, we make use of a key result from random matrix theory for performing integration over the functions of eigenvalues of a random matrix such that if $g(\mathbf{A}) = \sum_{i=1}^{n} g(\lambda_i)$ then $\text{E}\{g(\mathbf{A})\} = \int_{-\infty}^{\infty} g(\lambda) \text{f}(\lambda) d\lambda$ where $\text{f}(\lambda)$ is the average density of the eigenvalues of the random matrix or the average normalized histogram of the eigenvalues of random matrix samples that is also valid for composite functions of $g(\mathbf{A})$.

It can be shown by the general properties of the orthogonal Hermite polynomials that the sum of means of eigenvalues of $\mathbf{R}_{k\acute{k}}$ is zero and also by averaging over the average density of eigenvalues $\text{f}(\lambda)$ in (10) of Theorem 1. Furthermore, by Harer-Zagier Recursion [7], if $\mathbf{A}_{n \times n}$ is an element of **SGRM** $(n, 1)$, then the expected traces of the even powers of $\mathbf{A}_{n \times n}$, $C(p, n) = \text{E}\{\text{Tr}[\mathbf{A}^{2p}]\}$, is related by the recursion: $C(p+1, n) = n.\frac{4p+2}{p+2} C(p, n) + \frac{p(4p^2-1)}{p+2} C(p-1, n)$ with $C(0, n) = n$ and $C(1, n) = n^2$. Hence, the sum of mean-squared values of eigenvalues of $\mathbf{R}_{k\acute{k}}$ in **SGRM** $(D, \frac{1}{L})$ is $\frac{D^2}{L}$ via Harer-Zagier Recursion. Thus, denoting the factor *per-chip diversity order*, $\alpha = \frac{D}{L} \simeq \frac{T_m}{T_b} \ll 1$, that unitlessly measures the time-dispersivity of the channel[1] with respect to the bit signalling period and letting $K - 1 \simeq K$ as K gets large , the total MAI plus noise variance can be derived as:

$$\begin{aligned} &\sigma^2_{\text{MAI+n}} = \\ &\sum_{\acute{k}=2}^{K} \sum_{d=1}^{M_d D} \text{E}\left\{\Phi_{k\acute{k}d}\right\} \text{E}\left\{\left|\lambda_{k\acute{k}d}\right|^2\right\} \text{E}\left\{\left|h_{\acute{k}d}\right|^2\right\}. \\ &\text{NFR}.\text{SNR}_\text{b} \\ &+E\left\{\mid n_k^{(\text{RAKE})} \mid^2\right\} \\ &= \frac{M_d \beta \alpha}{2} L(M_b)(D+1)\text{NFR}.\text{SNR}_\text{b} + M_d D, \end{aligned} \tag{8}$$

where $L(M_b) = \text{E}\left\{\Phi_{k\acute{k}d}\right\}$ is the average beamforming power loss that quanitifies the average power loss of the dth path of $\acute{k}$th user due to beamforming.

Up-to-date in the literature, average beamforming loss is addressed and used, however in many cases numerical approximations through computer simulations are employed instead of analytical expressions for it as in [8]. In the following corollary, an analytic expression for the average beamforming power loss under uniformly distributed DOAs as a function of beamforming antennas is presented:

Corollary 1. In a uniform linear antenna array of M_b antennas with interelement spacing $d = \frac{\lambda_c}{l}$, the average beamforming power loss induced on a signal at DOA θ_j by the normalized array response steered towards DOA θ_i to achieve maximal-SIR, where the DOAs are assumed to be i.i.d. uniformly distributed between $[-\pi, \pi]$ radians with respect to the array normal, is analytically given as a function of the number of beamforming antennas by:

$$\begin{aligned} &L(M_b) = \\ &\text{E}\left\{\left|\mathbf{a}^H(\theta_i)\mathbf{a}(\theta_j)\right|^2\right\} = \frac{1}{M_b} \sum_{m=-M_b+1}^{M_b-1} J_0^2\left(\frac{2\pi m}{l}\right), \end{aligned} \tag{9}$$

[1] In this situation, the system operating point is uniquely defined by the load factor, diversity order, per-chip diversity order (normalized channel spread factor), near-far ratio and bit signal-to-noise ratio while the number of users and processing gain are hidden variables. As processing gain and consequently diversity order are increased , the number of users is also assumed to be increased to keep the load factor fixed.

where $J_0(x)$ is the Bessel function of the first kind of zeroth order.

Proof: The average beamforming power loss is given by:

$$L(M_b) = \mathrm{E}\left\{\left|\mathbf{a}^H(\theta_i)\mathbf{a}(\theta_j)\right|^2\right\} = \frac{1}{4\theta_s^2 M_b} \int_{-\theta_s}^{\theta_s}\int_{-\theta_s}^{\theta_s}\left|\mathbf{a}^H(\theta_i)\mathbf{a}(\theta_j)\right|^2 d\theta_i d\theta_j, \quad (10)$$

where the one-sided angular spread is θ_s, and without going into the details of the initial algebra, (10) can be put into form:

$$L(M_b) = \frac{1}{\theta_s^2 M_b}\sum_{m=-M_b+1}^{M_b-1}\left[\theta_s J_0\left(\frac{2\pi m}{l}\right) - \sin(2\theta_s m)\sum_{k=1}^{\infty}\frac{J_{2k}\left(\frac{2\pi m}{l}\right)}{k}\right]^2, \quad (11)$$

since:

$$\cos(x\sin(\theta)) = J_0(x) + 2\sum_{n=1}^{\infty} J_{2n}(x)\cos(2n\theta),$$
$$\sin(x\sin(\theta)) = 2\sum_{n=1}^{\infty}(-1)^n J_{2n}(x)\cos(2n\theta), \quad (12)$$

where $J_n(x) = \sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{x}{2}\right)^{2k+n}}{k!\Gamma_c(n+k+1)}$ is the Bessel function of the first kind of nth order [9]. Thus, setting $\theta_s = \pi$ in (11), the average beamforming power loss as a function of M_b is given by:

$$L(M_b) = \mathrm{E}\left\{\left|\mathbf{a}^H(\theta_i)\mathbf{a}(\theta_j)\right|^2\right\} = \frac{1}{M_b}\sum_{m=-M_b+1}^{M_b-1} J_0^2\left(\frac{2\pi m}{l}\right), \quad (13)$$

◊

Based on these characterizations for the terms in the SIR definition of (7), we are now ready to evaluate the average bit error probabilities. Via the Gaussianity of the total MAI plus noise components, the conditional bit error rate of the kth user conditioned on the eigenvalues of the cross-correlation matrix $\mathbf{R}_{kk}$ and the magnitude-squared distribution of the channel can be written as:

$$\mathrm{P}_{b,k} = Q\left(\sqrt{2\mathrm{SIR}_k^{(\mathrm{RAKE})}}\right), \quad (14)$$

due to coherent BPSK modulation [10] where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty}\exp(-\frac{t^2}{2})dt$ is the complementary Gaussian error function and $\mathrm{SIR}_k^{(\mathrm{RAKE})}$ can be equivalently written as the product of the random variables and a constant as $\mathrm{SIR}_k^{(\mathrm{RAKE})} = C_r\lambda^2 r$ via the averaging tool over the eigenvalues of random matrices. Here, the constant $C_r = \frac{\frac{1}{L^2}\mathrm{SNR_b}}{\sigma^2_{\mathrm{MAI+n}}}$ after taking the $\frac{1}{L}$ factors from $\mathbf{R}_{kk}$ out, λ is a central χ^2 distributed rv of $2L$ degrees of freedom with individual component variances of $\frac{1}{2}$ having pdf $\mathrm{f}(\lambda) = \frac{1}{\Gamma_c(L)}\lambda^{L-1}\exp(-\lambda)$ for $\lambda \geq 0$ with $\Gamma_c(n) = \int_0^{\infty} t^{n-1}\exp(-t)dt$ being the complete Gamma function, and r is the magnitude-squared distribution of the channel coefficients. Thus, under unity path power Rayleigh fading, r is a central χ^2 distributed rv of $2M_dD$ degrees of freedom with individual component variances of $\frac{1}{2}$ having pdf $\mathrm{f}(r) = \frac{1}{\Gamma_c(M_dD)}r^{M_dD-1}\exp(-r)$ for $r \geq 0$, while under unity path power Ricean fading, r is a non-central χ^2 distributed rv of $2M_dD$ degrees of freedom having pdf $\mathrm{f}(r) = \frac{1}{\sigma^2}\left(\frac{r}{\sigma^2}\right)^{M_dD-1}\exp\left(-\frac{r+s^2}{\sigma^2}\right)I_{M_dD-1}\left(\frac{2\sqrt{r}s}{\sigma^2}\right)$, where $s^2 = \sum_{i=1}^{M_dD}|\mu_i|^2 = M_dD(1-\sigma^2)$ is the non-centrality parameter and $I_n(x) = \sum_{k=0}^{\infty}\frac{\left(\frac{x}{2}\right)^{2k+n}}{k!\Gamma_c(n+k+1)}$ is the modified Bessel function of the first kind of nth order. Then, the unconditional average bit error probability is obtained by averaging $\mathrm{P}_{b,k}$ over the pdfs of λ and r as:

$$\mathrm{P}_{b,k}^{(\mathrm{av})} = \int_0^{\infty}\int_0^{\infty} Q\left(\sqrt{2C_r\lambda^2 r}\right)\mathrm{f}(\lambda)\mathrm{f}(r)d\lambda dr, \quad (15)$$

Integrals of the form as in (15) is in general hard to take since the argument of the function appears in the lower limit of the integral. To facilitate our derivation, we make use of an exact alternative form of the Q-function proposed by Craig [11] in terms of an exponential integral with trigonometric argument as:

$$Q(x) = \frac{1}{\pi}\int_0^{\pi/2}\exp\left(-\frac{x^2}{2\sin^2(\phi)}\right)d\phi, \quad (16)$$

that is valid for all values of the argument of x.

Let's first focus on the Rayleigh fading case. Via (15) and (16), the average probability of bit error under Rayleigh fading can be written after some algebraic arrangement in terms of a triple integral as:

$$\mathrm{P}_{b,k}^{(\mathrm{av,Ray})} = \frac{1}{\pi\Gamma_c(L)\Gamma_c(M_dD)}\int_0^{\infty}\int_0^{\infty}\int_0^{\pi/2}\exp\left(-\frac{C_r\lambda^2 r}{\sin^2(\phi)}\right) .\lambda^{L-1}\exp(-\lambda)r^{M_dD-1}\exp(-r)d\lambda dr d\phi, \quad (17)$$

and taking the integral first with respect to r that is a Gamma integral yields:

$$\mathrm{P}_{b,k}^{(\mathrm{av,Ray})} = \frac{1}{\pi\Gamma_c(L)}\int_0^{\infty}\lambda^{L-1}\exp(-\lambda)\int_0^{\pi/2}\frac{\sin^{2M_dD}(\phi)}{(\sin^2(\phi)+C_r\lambda^2)^{M_dD}}d\phi d\lambda, \quad (18)$$

since $\int_0^{\infty} t^{n-1}\exp(-at)dt = \frac{\Gamma_c(n+1)}{a^{n+1}}$ [12]. The trigonometric integral over ϕ in (18) has no known closed-form solution, however a quite accurate and useful approximation on the average bit error probability that leads to further tractability can be obtained by setting $\sin^2(\phi) = 1$ in the denominator of the integrated function over $[0, \frac{\pi}{2}]$. Hence, we obtain a tractable approximate expression on the average bit error probability over Rayleigh fading as:

$$\mathrm{P}_{b,k}^{(\mathrm{av,Ray})} \cong \frac{C(2M_dD, M_dD)}{2^{2M_dD+1}\Gamma_c(L)}\int_0^{\infty}\frac{\lambda^{L-1}\exp(-\lambda)}{(1+C_r\lambda^2)^{M_dD}}d\lambda, \quad (19)$$

since $\int_0^{\pi/2} \sin^{2n}(\phi)d\phi = \frac{\pi}{2}\frac{C(2n,n)}{2^{2n}}$ [25] where $C(n,r) = \frac{n!}{r!(n-r)!}$ is the combinatorial function.

Finally, to evalute the quite complex integral over λ parametrized by $L, M_d D$ and C_r simultaneously, we employ nth order polynomial least-squares optimal Gauss-Laguerre quadrature numerical integration that is good for the integrals of the type $\int_0^\infty f(x)\exp(-x)dx$ as:

$$\mathrm{I}_{\mathrm{GL}}^{(\mathrm{Ray})} = \int_0^\infty \frac{\lambda^{L-1}\exp(-\lambda)}{(1+C_r\lambda^2)^{M_d D}} d\lambda \simeq \sum_{i=1}^{n} w_i \frac{x_i^{L-1}}{\left(1+C_r x_i^2\right)^{M_d D}}, \tag{20}$$

where x_i are the roots of the nth order orthogonal Laguerre polynomial and w_i are the corresponding optimal weights that can be found in [13]. Thus, we conclude that the average probability of bit error of kth user with hybrid reception space-time RAKE receiver over Rayleigh fading is approximately given by:

$$\mathrm{P}_{b,k}^{(\mathrm{av,Ray})} \cong \frac{C(2M_d D, M_d D)\mathrm{I}_{\mathrm{GL}}^{(\mathrm{Ray})}}{2^{2M_d D+1}\Gamma_{\mathrm{c}}(\frac{D}{\alpha})}, \tag{21}$$

Similarly without going into the algebraic details, by lower-bounding the integral over ϕ, the average probability of bit error of kth user with hybrid reception space-time RAKE receiver over Ricean fading can be derived approximately as:

$$\mathrm{P}_{b,k}^{(\mathrm{av,Rice})} \cong \frac{\exp\left(-\frac{M_d D(1-\sigma^2)}{\sigma^2}\right)}{2^{2M_d D+1}\Gamma_{\mathrm{c}}(\frac{D}{\alpha})} \sum_{k=0}^{\infty} \frac{\left(\frac{M_d D(1-\sigma^2)}{\sigma^2}\right)^k C(2(M_d D+k), M_d D+k)\mathrm{I}_{\mathrm{GL}}^{(\mathrm{Rice})}(k)}{2^{2k}.k!}, \tag{22}$$

where:

$$\mathrm{I}_{\mathrm{GL}}^{(\mathrm{Rice})}(k) = \int_0^\infty \frac{\lambda^{L-1}\exp(-\lambda)}{\left(1+\sigma^2 C_r\lambda^2\right)^{M_d D+k}} d\lambda, \tag{23}$$

is to be computed by Gauss-Laguerre quadrature numerical integration.

4. NUMERICAL RESULTS and DISCUSSION

We present a numerical Monte-Carlo simulation study for the validation of the derived approximate theoretical average bit error probability expressions with the hybrid reception space-time RAKE receiver in (21) and (22) over Rayleigh and Ricean fading channels in Fig. 3. at a certain operating point with total number of antennas $M = 4$. In simulations, besides other parameters indicated in the caption, the mobile speed is 30 km/h, carrier frequency is 1800 MHz, interelement spacing $d_m = \frac{\lambda_c}{4}$ and $K \times D$ DOAs are choosen to be i.i.d. uniformly distributed between $[-\pi, \pi]$ radians. The simulation results obtained by 10^4 averaging Monte-Carlo runs at each SNR point closely match the derived approximate theoretical expressions via 32-point Gauss-Laguerre quadrature numerical integration with small mismatch on the error floors especially when M_d is high due to the errors from

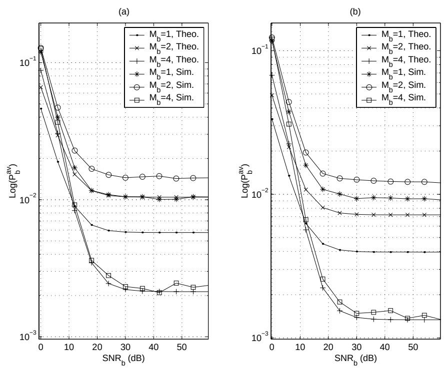

Figure 3: Comparison of analytic and simulated average bit error probabilities versus average bit SNR per path of desired user SNR_b at $\beta = 1, \alpha = 0.1, D = 2, \mathrm{NFR} = 5\mathrm{dB}$ for different antenna partionings at $M = 4$, (a): Rayleigh fading, (b): Ricean fading with variance of channel coefficients 0.5.

approximations and numerical integration used in the derivation of the approximate theoretical expressions. Thus, this close match proves the usability of the derived expressions in the analysis of our problem of optimal antenna partioning strategies between spatial diversity reception and beamforming.

We present the average bit error probabilities over Rayleigh and Ricean fading versus the frequency diversity order D in Fig. 4 and as expected, average bit error probabilities strictly decrease with increased frequency diversity order[2]. At this system operating point, for both Rayleigh and Ricean fading cases, choosing 4 beamforming antennas and 2 spatial diversity groups is optimal in terms of average bit error rate while a very small gain can be obtained for flat fading case at $D = 1$ by choosing $M_b = 1$. In addition, choosing $M_b = 1$ always results in lower average bit error probability with respect to $M_b = 2$ case for all diversity orders. Furthermore, the average error probability with $M_b = 8$ results in higher average error probability with respect to $M_b = 1$ and $M_b = 2$ at low frequency diversity orders due to low effective diversity order $D_{\mathrm{eff}} = M_d \times D$, however crossovers occur as D increases and the average bit error probability at $M_b = 8$ gets lower than that of both $M_b = 1$ and $M_b = 2$ cases.

We further present the average bit error probabilities over Rayleigh and Ricean fading versus the average bit SNR per path of desired user in Fig. 5. Since at low average bit SNR, it is vital to increase the effective received signal-to-noise ratio via diversity, the average bit error probabilites gets worse as M_b is increased at low average bit SNR per path. On the other hand, as average bit

[2]The truncation of $M_b = 1$ curve at $D = 4$ for Ricean fading case is due to the numerical range insufficiency in MATLAB for the computation of the expression, however the trend is the same.

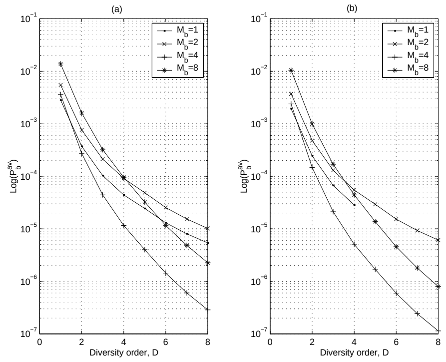

Figure 4: Average bit error probability versus diversity order D at $\beta = 1, \alpha = 0.1, \mathrm{SNR}_b^1 = 15\mathrm{dB}, \mathrm{NFR} = 0\mathrm{dB}$ for different antenna partionings at $M = 8$, (a): Rayleigh fading, (b): Ricean fading with variance of channel coefficients 0.5.

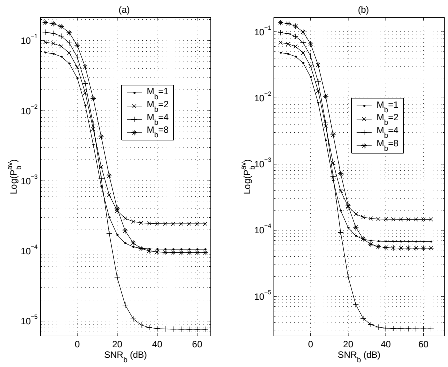

Figure 5: Average bit error probability versus average bit SNR per path of desired user SNR_b in dB at $D = 2, \alpha = 0.1, \beta = 1, \mathrm{NFR} = 0\mathrm{dB}$ for different antenna partionings at $M = 8$, (a): Rayleigh fading, (b): Ricean fading with variance of channel coefficients 0.5.

SNR per path increases, the effect of beamforming comes into play, crossovers occur and the order of antenna partionings in terms of descending average bit error probabilites for the error floors at high average bit SNR per path reads as $M_b = 2$, $M_b = 1$, $M_b = 8$ and $M_b = 4$ respectively at this operating point yielding the $M_b = 4/M_d = 2$ as the optimal antenna partioning at high average bit SNR per path.

5. CONCLUSIONS

In this paper, based on a finite-size system model for random spreading analysis, we derived and analyzed the average bit error rates of a hybrid diversity-combining/beamforming space-time RAKE receiver over Rayleigh/Ricean fading time-varying multipath BPSK-CDMA channels. The analysis of the results presents importance for dechiphering the optimal antenna partioning strategies between spatial diversity reception and beamforming functionality in conjunction with the system operating point defined by the key system parameters.

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