Antenna Diversity Techniques for a Single Carrier System with Frequency Domain Equalization – An Overview

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Abstract: **In this work we investigate the possibilities of combining diversity techniques with a Single Carrier System with Frequency Domain Equalization (SC/FDE), as future transmission concepts will also be judged by their possibilities to be combined with multiple antennas. Concepts of multiple antennas allow either a significant performance gain through diversity or significant higher capacity based on spacial multiplexing. In this contribution we will point out the excellent possibilities of SC/FDE to be combined with diversity techniques, as the process of the equalization itself can be combined with receive diversity as well as with space time block (STBC) decoding. This combination gives a reduced signal processing complexity as well as the expected enormous performance gain. Especially the concept of STBC allows an advantageous combination with SC/FDE as both are based on a blockwise processing that can be formulated advantageously in the frequency domain. Our simulation results are based on the parameters of the IEEE 802.11a standard.**

1. Introduction

The ambition for extreme high data rates as well as high quality of service make new concepts for communication systems indispensable. Future physical layer concepts will have to ensure better performance, higher data rates as well as a high spectral efficiency. Another decisive factor is the signal processing complexity, especially for mobile applications.

Concepts of multiple antennas allow to fulfill several of the above mentioned criteria in a superior way. While on the one hand multiple antenna systems allow to enhance the spectral efficiency tremendously by spacial multiplexing, they allow to improve the performance by diversity on the other hand. From that point of view, the question is obvious, if there are transmission concepts (modulations schemes/physical layer concepts) that can be combined in a more profitable way with concepts of multiple antennas than others - concepts that combine the best performance with a low signal processing complexity.

Especially the wireless indoor radio communication has to face problems, as the time dispersion caused by multipath propagation makes powerful equalization concepts necessary to guarantee an acceptable performance, at the cost of a high signal processing complexity. Concepts that implement this equalization in the frequency domain are able to fulfill both criteria, good performance and reduced implementation effort. The main advantage over a time domain equalization is that the complexity for an FFT based frequency domain equalization grows only slightly more than linear with the bit rate, while the complexity grows at least quadratically with the bit rate for traditional time domain equalizers [1]. Concepts that implement an efficient FFT based frequency domain equalization are OFDM (Orthogonal Frequency Division Multiplexing) and SC/FDE (Single Carrier System with Frequency Domain Equalization). While the concept of OFDM is used in different standards, the concept of SC/FDE is rarely investigated, although this approach combines the advantages of OFDM and single carrier transmission [2][3][4].

This paper is organized as follows: In the next section we will introduce the concept of the investigated SC/FDE system. The following chapter deals with receive diversity and how this can be combined with the SC/FDE concept. Possible performance gains and the required implementation effort will be discussed. The same discussion will be carried out for transmit diversity (STBC). Finally an overall perception that evaluates the applicability of SC/FDE for diversity techniques will conclude this work.

2. The Concept of SC/FDE

In an SC/FDE system the received signal must be transferred to the frequency domain (by means of an efficient FFT operation), where the equalization takes place. Similar to OFDM, a cyclic prefix is inserted between successive blocks in the transmitter in order to mitigate interblock interference (IBI) [5]. To prevent this IBI the duration of the guard period T_G must be longer than the duration of the channel impulse response T_h. Due to the cyclic extension, the linear convolution of one cyclically extended transmitted block and the channel impulse response *h(t)* appears as a circular convolution corresponding to the frequency domain relation

$$R(nf_0) = H(nf_0)S(nf_0) + N(nf_0) \quad (1)$$

for n∈Z and f0=1/TFFT. Here the functions R(f), S(f) and H(f) are related to the time domain signals r(t) (one period of the received data block), *s(t)* (the original, non cyclically extended block) and *h(t)* by the continuous Fourier transform. *N(f)* is the Fourier transform of the additive noise. Figure 1 shows the transmitted data structure, which consists of the data sequence containing N symbols and the sequence of the cyclic prefix with N_G symbols. The duration of a processed block is $T_{FFT}=NT$ and the duration of the cyclic prefix is $T_G=N_GT$.

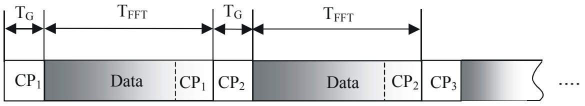

Figure 1 Transmitted data structure in a SC/FDE.

In this contribution a fractionally spaced equalizer is implemented that allows a low complexity implementation as well as a compact mathematical description that includes multiple antennas. The received signal is sampled with twice the symbol rate, which is sufficient for band-limited pulses and the sampled sequence of each received and processed block is split into its polyphases $r_1(k)$ and $r_2(k)$, were both sequences contain N samples that correspond to symbol rate sequences. The N-point FFT transformed sequences $R_1(nf_0)$ and $R_2(nf_0)$ are then multiplied (equalized) with the equalizer phases $E_1(nf_0)$ and $E_2(nf_0)$.The equalized signal

$$Y_D^T(nf_0)=E_1(nf_0)X_1(nf_0)+E_2(nf_0)X_2(nf_0) \tag{2}$$

is finally transformed back to the time domain by an N-point IFFT. The equivalence of classical structure and the polyphase structure is documented in Figure 2.

Two equalization criteria are discussed in this contribution - the Zero Forcing (ZF) criterion and the Minimum Mean Square Error (MMSE) criterion. The polyphase representation of the ZF equalizer has been developed in [6] and is given in equation (3)

$$\mathbf{E}_{ZF}(f)=\frac{\hat{\mathbf{F}}^H(f)}{\sum_{i=1}^{2}\left|\hat{F}_{i,}(f)\right|^2}=\frac{\hat{\mathbf{F}}^H(f)}{\left\|\hat{\mathbf{F}}(f)\right\|^2} \tag{3}$$

where the superscript ^{H} denotes Hermitian transpose. In this equation

$$\hat{\mathbf{F}}(f)=\left[\hat{F}_1(f),\hat{F}_2(f)\right]^t \tag{4}$$

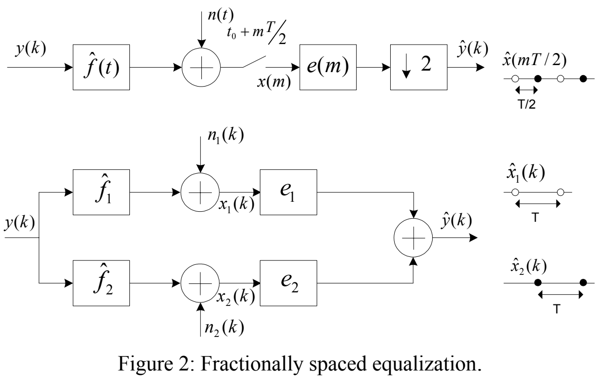

Figure 2: Fractionally spaced equalization.

describes the fourier transformed, channel distorted transmitted pulse that has to be derived by pilot aided channel estimation procedures. It is essential to mention that in the case of a blockwise transmission based on cyclic prefix, an optimal equalizer of infinite length is implemented. From the analytical expression of the equalizer it gets obvious that the nominator of equation (3) characterizes a matched filter that maximizes the SNR for the two polyphases, while the denominator describes the ZF equalizer itself that forces an ISI free detection. It is well known that the ZF equalizer suffers from noise amplification if the channel transfer function shows deep spectral fades. An equalizer that avoids this problem by compromising the noise amplification and the ISI reduction is the Minimum Mean Square Error equalizer (MMSE). The optimum infinite-length MMSE equalizer is given by

$$\mathbf{E}_{D,MMSE}(f)=\frac{\hat{\mathbf{F}}_D^H(f)}{\sum_{i=1}^{2}\left|\hat{F}_{i,D}^T(f)\right|^2+\sigma_n^2/\sigma_d^2}. \tag{5}$$

Here it is assumed that the input data sequence *d(k)* is uncorrelated with variance σ_d^2 and the noise in the two branches is white with variance σ_n^2. Compared to the ZF equalizer the additive term σ_n^2/σ_d^2 in the denominator protects against infinite noise amplification since the denominator is always strictly positive. The ZF equalizer is a sub-optimum equalizer in the case of strong multipath propagation but it will be shown that for multiple antennas this is sufficient.

3. Receive Diversity for SC/FDE

Different concepts as switched antenna diversity, selection diversity, equal gain combining, or maximum ratio combining are well known. The most powerful but also the concept with the highest implementation effort is maximum ratio combining. There, the signal that is received due to multiple antennas (M) is added and additionally weighted in an optimal way to enhance the actual SNR. The optimal equalizer for multiple antennas can be defined as follows

$$\mathbf{E}_{ZF}(f)=\frac{\hat{\mathbf{F}}^H(f)}{\left\|\hat{\mathbf{F}}(f)\right\|^2}=\frac{\hat{\mathbf{F}}^H(f)}{\sum_{j=1}^{M}\sum_{i=1}^{2}\left|\hat{F}_{i,j}(f)\right|^2}, \tag{6}$$

were

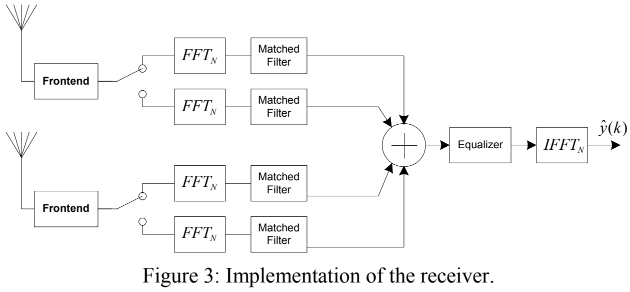

Figure 3: Implementation of the receiver.

$$\hat{\mathbf{F}}(f) = \left[\hat{F}_{1,1}(f), \hat{F}_{1,2}(f), \hat{F}_{2,1}(f), \hat{F}_{2,2}(f),\right]', \quad (7)$$

for the case of M=2. The MMSE Equalizer is defined equivalently. Notice that the equalization term itself is independent of the individual antenna path or polyphase and can therefore be separated from the matched filter part. This allows the following implementation as shown in Figure 3. The advantages of this implementation are as follows: a) The matched filter for every path implements the optimal weighting and guarantees an optimal performance. b) The equalization has to be carried out only once for every block instead of for every path.

An additional advantage is that a simple N-point IFFT is necessary to transform the received and equalized blocks back to the time domain.

The performance gain of receive diversity is based on two different aspects:

a) Antenna Gain: Due to the reception of the same sent sequence on every antenna, the received signal power is improved. This enhances the average output SNR – based on the fact that the received sequences due to different antennas are correlated, while the noise sequences are uncorrelated. In the AWGN case this leads to gain M for M receive antennas

$$G(M)_{AWGN} = \frac{SNR_{MRx}}{SNR_{1Rx}} = M \quad (8)$$

Doubling the number of antennas leads to a gain of *3* dB. Equation (8) is still valid for multipath conditions, if the investigated channel transfer functions show unity average power (that r

b) Diversity gain: The output-signal of every matched filter and its addition can simply be described as the quadratical summation of several channel transfer functions as given in equation (9).

$$A(f) = \sum^{M} \left|H(f)\right|^{2} \quad (9)$$

Due to this addition, deep spectral fades of single paths are suppressed under the assumption of uncorrelated channel transfer functions. $A(f)$ follows a chi-square distribution with a mean value of $E(A)=M$. That means that the variance of $A(f)$ about its mean value diminishes for increasing M over the frequency band. Finally we can conclude that for a high number of antennas [7]

$$G_{AWGN}(M) \cong E[A] = M \quad (10)$$

This statement was anticipated in equation

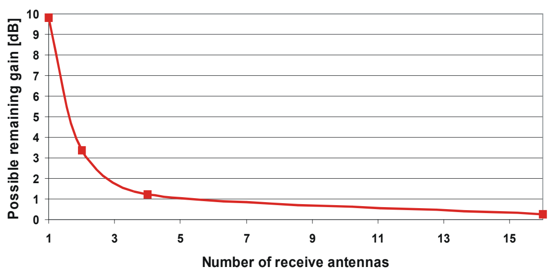

Figure 4: Possible remaining gain at a BER of 10-4.

(8) already. A high number of receive antennas leads to an almost flat behaviour of the overall transfer function and prevents additional gain due to diversity. Figure 4 clearly documents this fact. There, the possible performance gain based on diversity that is limited by the matched filter bound is shown as a function of the number of antennas (for this diagram, the constant gain of M based on the antenna gain has been subtracted). It is obvious that already for four antennas the possible additional gain based on diversity is less than *1 dB* and will in general not justify additional antennas. To conclude this consideration, the bit error behaviour for one, two and four receive antennas is given Figure 5. The simulation results are based on 100 randomly chosen indoor radio channel snapshots based on the IEEE standardization model (tapped delay line, each tap with random uniformly distributed phase, Rayleigh distributed magnitude and with the power delay profile decaying exponentially). The BER curves prove the expected behaviour. Besides the tremendous gain it gets obvious that the performance gain of an MMSE equalizer in comparison to a simple ZF equalizer is not more than 1 dB for two antennas and vanishes completely for four antennas. The simulations additionally document that the performance loss in comparison to the matched filter bound is not more than 1 dB for four antennas. This makes other equalization concepts as decision feedback equalization or maximum likelihood sequence estimation needless, as the additional implementation effort will not justify the possible performance gain.

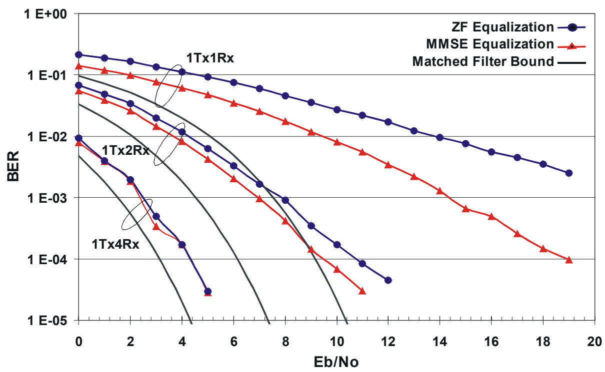

Figure 5: BER for receive diversity.

Of course it is to mention that a practical implementation of a higher number of antennas than two is critical, especially when talking of mobile and handheld applications. From that point of view it is of interest to investigate the possible improvement due to transmit diversity.

4. Transmit Diversity based on Space-Time block coding.

Orthogonal designs of space time codes provide an efficient means to generate codes that achieve the full diversity gain. For two transmit antennas, the orthogonal design is known as the Alamouti scheme, originally formulated for the flat fading case. In [9] this concept was extended for the case of frequency selective fading by Al-Dahir. While the concept of Alamouti describes a symbol by symbol coding, the concept of Al-Dahir implements a block by block coding/processing, that is combined with the concept of SC/FDE with less additional effort. Figure 6 shows the block format of STBC for SC/FDE and Table 1 gives the coding scheme. (in the following the expression $b_m^N(i)$ stands for the *ith* of *N* symbols of the *mth* block). Notice that besides a simple complex conjugation of every second processed block, the order is changed that allows an advantageous implementation of the decoding in the frequency domain. The advantage of this changed order gets obvious if we take a look at the Fourier transformation of the coded blocks.

$$\begin{aligned} &b_m^N(i) \bullet\!-\!\circ\, B_m^N(k) \\ &b_m^{N*}((-i)_N) \bullet\!-\!\circ\, B_m^{N*}(k) \end{aligned} \qquad (11)$$

In this equation $(-i)_N$ stands for *(-i) modulo N.*

Based on the blockwise processing, two received blocks can be written as follows.

$$\begin{aligned} R_1^N(k) &= H_1^N(k)\frac{B_1^N(k)}{\sqrt{2}} + H_2^N(k)\frac{B_2^N(k)}{\sqrt{2}} + N_1 \\ R_2^N(k) &= -H_1^N(k)\frac{B_2^N(k)^*}{\sqrt{2}} + H_2^N(k)\frac{B_1^N(k)^*}{\sqrt{2}} + N_2 \end{aligned} \qquad (12)$$

The factor $\sqrt{2}$ considers that the sent energy over the antennas is the same as for the system with one antenna.

Receiving these distorted and coded blocks, they have to be decoded based on the following scheme:

Datablock	1. Antenna	2. Antenna
b_1^N	---	---
b_2^N	$b_1^N(i)$	$b_2^N(i)$
b_3^N	$-b_2^{N*}((-i)_N)$	$b_1^{N*}((-i)_N)$
b_4^N	$b_3^N(i)$	$b_4^N(i)$
b_5^N	$-b_4^{N*}((-i)_N)$	$b_3^{N*}((-i)_N)$
...	...	...

Table 1: Coding scheme.

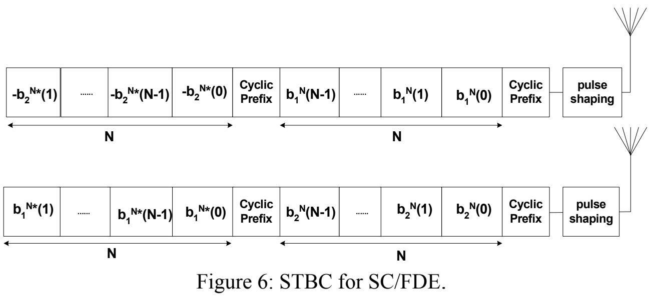

Figure 6: STBC for SC/FDE.

$$\begin{aligned} \hat{B}_1^N(k) &= H_1^{N*}(k)R_1^N(k) + (H_2^{N*}(k)R_2^N(k))^* \\ \hat{B}_2^N(k) &= H_2^{N*}(k)R_1^N(k) - (H_1^{N*}(k)R_2^N(k))^* \end{aligned} \qquad (13)$$

From equation (13) the main advantage of a combination of SC/FDE and STBS gets abvious again. The main part of the decoding is the multiplication with *H(k)** – that characterizes the matched filter that is implemented anyway.

A decoded block of symbols can finally be calculated as follows:

$$\begin{aligned} \hat{B}_1^N(k) &= \frac{|H_1^N(k)|^2 + |H_2^N(k)|^2}{\sqrt{2}} B_1^N(k) + H_1^{N*}(k)N_1 + H_2^N(k)N_2^* \\ \hat{B}_2^N(k) &= \frac{|H_1^N(k)|^2 + |H_2^N(k)|^2}{\sqrt{2}} B_2^N(k) + H_2^{N*}(k)N_1 - H_1^N(k)N_2^* \end{aligned} \qquad (14)$$

Based on the received and decoded block, the SNR at every frequency point can be calculated as

$$SNR(k) = \frac{|H_1^N(k)|^2 + |H_2^N(k)|^2}{2}\frac{\rho_s^2}{\rho_n^2} \qquad (15)$$

were ρ_S^2 defines the power of the signal and ρ_n^2 defines the power of the additive noise. This leads to the final SNR of the entire block

$$SNR = \sum_{k=0}^{N-1} SNR(k) = \frac{E_{H1} + E_{H2}}{2}\frac{\rho_s^2}{\rho_n^2} \qquad (16)$$

where *E* describes the energy of the channel transfer function. Equation (16) demonstrates that the concept of STBC based on Alamouti or Al Dahir leads to an averaging of the SNR of both channels – but not to an antenna gain due to multiple reception of the same signal as it is the case for receive diversity. The possible gain due to diversity is comparable for transmit as well as receive diversity. Comparable coding concepts of STBC exist for four and eight antennas [10],[11]. Figure 7 documents that the increase of transmit antennas leads to a convergence to the matched filter bound for one antenna. Using four transmit antennas, the performance loss in comparison to the case of AWGN is not more than *1 dB* anymore. Again it gets obvious that for more than two transmit antennas the difference between ZF and MMSE equalization is almost negligible. Nevertheless, in comparison to receive diversity, the possible gain is reduced by *6 dB* (see Figure 5).

Notice that the concept of block coding leads to a time delay of one block using two antennas and four blocks using four antennas that might be taken into account for time critical implementations. Additionally it is to mention that STBCs for more than two antennas will lead to a loss of data rate, because only three blocks of information are sent over four antennas in the investigated case.

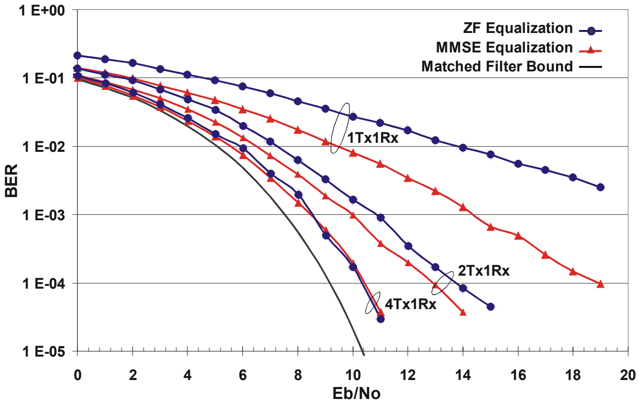

Figure 7: BER for STBC.

5. The combination of transmit and receive diversity

Finally we want to point out the advantages of a combined implementation of transmit and receive diversity. Figure 8 shows the possible implementation of transmit and receive diversity in the investigated system. The concept of coding is identical to a system containing several transmit antennas and only one receive antenna. The difference gets obvious at the receiver, where for every receive antenna an individual decoding unit has to be implemented and after this the equalization is carried out. Notice that matched filtering, optimal weighting and decoding is implemented in one. The *mth* block of the *jth* receive unit can be written as follows

$$\hat{B}^N_{m,j}(k) = B^N_m(k)\frac{1}{\sqrt{t}}\sum_{l=1}^{t}\left|H^N_{l,j}(k)\right|^2 + N_{\hat{B}_{m,j}} \cdot \quad (17)$$

In this equation $H_{k,j}$ describes the channel transfer function between transmit antenna l and receive antenna j. t describes the total number of transmit antennas. As every antenna receives the same information, a simple addition of the r receive paths implements the receive diversity

$$\hat{B}^N_m(k) = \sum_{j=1}^{r}\hat{B}^N_{m,j}(k) = B^N_m(k)\frac{1}{\sqrt{t}}\sum_{j=1}^{r}\sum_{l=1}^{t}\left|H^N_{l,j}(k)\right|^2 + N_{\hat{B}_m} \cdot \quad (18)$$

Without going into details we conclude that the SNR

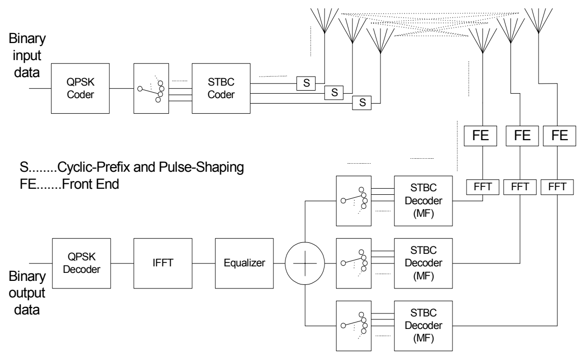

Figure 8: Concept of transmit and receive diversity for SC/FDE.

of this system is

$$SNR = \frac{1}{t}\sum_{j=1}^{r}\sum_{l=1}^{t} SNR_{l,j} \quad (19)$$

Again it gets obvious that receive diversity results in a summation of the average SNR while transmit diversity results in an averaging of the SNR. Figure 9 shows the BER due to transmit and receive diversity. For the 4Tx-4Rx system the performance corresponds almost to the case of AWGN using four receive antennas. The additional gain in comparison to simple receive diversity is about *1 dB* – at the cost of four transmit antennas. The most interesting concept, focusing an competitive implementation, is obviously the concept of 2Tx-2Rx. This system combines high performance gain with affordable implementation effort.

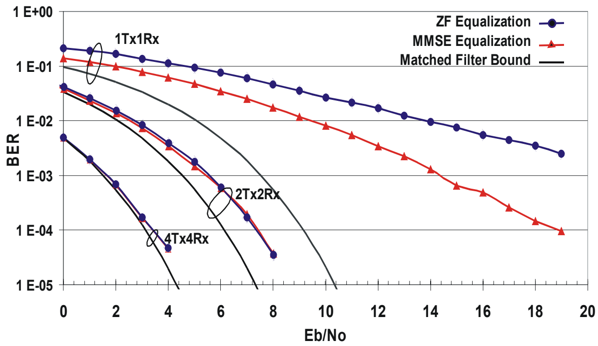

Figure 9: BER for transmit and receive diversity.

6. Conclusion

It can be concluded that diversity techniques allow tremendous gains in terms of BER. The highest profit due to diversity is reached already when using two receive or transmit antennas – that combines affordable implementation effort with significant better performance. Further more it has been demonstrated that a low complexity ZF equalizer is sufficient when using several antennas, although this equalizer is known as a suboptimal solution in the case of multipath conditions. The implementation effort of more powerful equalization concepts will not justify the additional performance gain. The most important statement of this investigations is that the concept of SC/FDE is not only a powerful candidate for future high rate communication systems, but can be combined with diversity techniques advantageously. The advantages of an SC/FDE system are its reduced implementation complexity combined with high performance. The combination of SC/FDE with diversity techniques enhances these advantages additionally, as the concept of the frequency domain equalization can be combined with the implementation of maximum ratio combining as well as STBC in an easy way.

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