Modeling the Mobile Radio Channel using Theory of Dynamics – First Derivations and Results –

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Abstract : **Traditionally, the mobile radio channel is modeled as a linear stochastic process using e.g. Rayleigh- or Rice-statistics for the amplitude and autocorrelation functions in terms of modeling the Doppler-power spectrum. It can be shown that this statistical modeling has some physical limits, e.g. at high bandwidths of wireless communication systems. For this reason, we introduce an alternative approach based on methods of the *theory of dynamics*, which overcomes these limitations. The aim is, to find a low-dimensional deterministic model consisting of a set of differential or difference equations instead of a high-dimensional stochastic model. Investigations of the mobile radio channel by algorithms adapted from *nonlinear dynamics* provide the advantage to distinguish between deterministic dynamical effects and stochastic effects. Therefore, in this paper, important analysis algorithms are introduced and applied to analyze simulated data from a ray-tracing simulation of a simple urban environment. The results are very promising that modeling the mobile radio channel based on the theory of dynamics is possible.**

1. Introduction

Not every data series which seems to be random is the output of a stochastic process. Often, there is an underlying deterministic process, causing this random looking signal. Probably, one of the best examples is playing dice. The output of this *process* is well known and easy to describe by a statistical probability-density function which is omni-distributed between one and six. Beyond this statistics, the motion of a dice can be expressed by differential equations. Thus, it is a deterministic process. The random looking output has its reason in the nonlinear nature of the related set of differential equations. That makes it sensitive to initial conditions. Every small change of angle or velocity during throwing the dice will cause an unpredictable result between one and six. This is called *deterministic chaos*. Nevertheless, many processes in physics or engineering with chaotic behavior are well characterized by statistical interpretations of their output. One example is the mobile radio channel.

Traditionally, the mobile radio channel is modeled to be a linear stochastic process, using e.g. Rayleigh- or Rice-statistics for the amplitude and autocorrelation functions in terms of modeling the Doppler-power spectrum. A well known method based on these assumptions is the WSSUS-model, which was first published by Bello [1].

Within these models, generally the mobile radio channel is characterized by the time-variant discrete complex lowpass impulse response

$$h(\tau,t)=\sum_{n=0}^{N-1} a_n(t)\cdot e^{j\alpha_n(t)}\cdot\delta(\tau-\tau_n(t)), \qquad (1)$$

where $a_n(t)\cdot e^{j\alpha_n(t)}$ is a complex time-variant random variable [2].

It is a realistic assumption that the impulse response of the physical channel in (1) consists of a very large number of N propagation paths, which are not equally spaced in the τ-domain (delay-time domain). Since the receiver has a limited bandwidth B and an own sample frequency $f_m = 1/\tau_m$, its $h_{Rx}(\tau,t)$ is different from the impulse response $h(\tau,t)$ of the physical channel as depicted in Figure 1 and Figure 2:

$$h_{Rx}(\tau,t)=\cdot \sum_{n=0}^{N-1} a_n(t)\cdot e^{j\alpha_n(t)}\cdot g_{LPF}(\tau-\tau_n(t))\cdot \sum_{m=1}^{M}\delta(\tau-m\cdot\tau_m) \qquad (2)$$

With the dependency $f(\tau)\cdot\delta(\tau-m\cdot\tau_m) = f(m\cdot\tau_m)\cdot\delta(\tau-m\cdot\tau_m)$ it follows that in τ-domain $h_{Rx}(\tau,t)$ is an equally spaced discrete function, where the amplitude of each Dirac-impulse is one sample of the superposition of the physical channel's impulse response shaped by the lowpass filter having a bandwidth B as depicted in Figure 2:

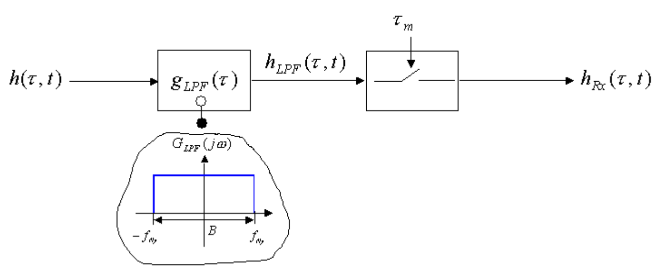

Figure 1: Simplified model of a receiver with a lowpass filter and a sampler; $g_{LPF}(\tau)$ is the impulse response of the filter, with bandwidth B and Nyquist-frequency f_{Ny}. $h(\tau,t)$ is the discrete impulse response of the physical channel. $h_{LPF}(\tau,t)$ is a continuous function which describes the impulse response after the LPF and $h_{Rx}(\tau,t)$ is a discrete impulse response reconstructed by the receiver.

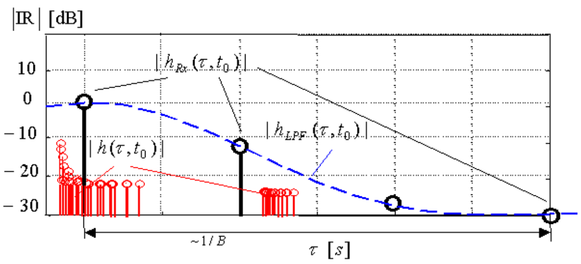

Figure 2: Typical example for the dependency between the different impulse responses (IR): $h(\tau,t)$, $h_{LPF}(\tau,t)$ and $h_{Rx}(\tau,t)$ at a fixed observation time t_0, where the channel can be considered to be time-invariant.

$$h_{Rx}(\tau,t) = \left. \sum_{m=1}^{M} \left[\delta(\tau - m\cdot\tau_m)\cdot\left[\sum_{n=0}^{N-1} a_n(t)\cdot e^{j\alpha_n(t)}\cdot g_{LPF}(\tau-\tau_n(t)) \right] \right] \right|_{\tau=m\cdot\tau_m} \quad (3)$$

$$= \sum_{m=1}^{M}\sum_{n=0}^{N-1} a_n(t)\cdot e^{j\alpha_n(t)}\cdot g_{LPF}\left(m\cdot\tau_m - \tau_n(t)\right), \quad (4)$$

where $\tau_m \le 1/B$ to fulfill Nyquist's sampling theorem [2].

It is obvious from equation (4) that the number of propagation paths (uncorrelated random variables), the sample $a_n(t)\cdot e^{j\alpha_n(t)}\cdot g_{LPF}\left(m\cdot\tau_m - \tau_n(t)\right)$ consists of, is getting smaller while the bandwidth is increasing. In practice, this number can also be reduced due to the use of high gain antennas for spatial filtering purposes. Additionally, keeping in mind that the term $a_n(t)\cdot e^{j\alpha_n(t)}$ can not be considered to be uncorrelated for different *n* in every case (for example if propagation paths are reflected at neighboring walls [16]), the central limit theorem [3] runs the risk to be harmed.

In this case, we do not have a Rice- or Rayleigh-amplitude statistic anymore, because we do not have Gaussian distributions in the real and imaginary part. The channel's characteristic becomes more and more deterministic.

All these aspects motivated us to look for alternative models for the mobile radio channel.

2. Recunstruction of dynamics

A very important issue in the study of dynamical systems is that of *model reconstruction* or *reconstruction of dynamics*. Specifically, given a set of data, one wishes to construct a dynamical model in the form of a set of differential or difference equations as defined in (5) and (6) which replaces the process producing the data:

$$\dot{y} = F(y) \quad \Leftrightarrow \quad \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \vdots \\ \dot{y}_{d_L} \end{bmatrix} = \begin{bmatrix} F_1(y_1, y_2, \ldots, y_{d_L}) \\ F_2(y_1, y_2, \ldots, y_{d_L}) \\ \vdots \\ F_{d_L}(y_1, y_2, \ldots, y_{d_L}) \end{bmatrix} \quad (5)$$

$$y^{(n+1)} = \phi(y^{(n)}) \Leftrightarrow \begin{bmatrix} y_1^{(n+1)} \\ y_2^{(n+1)} \\ \vdots \\ y_{d_L}^{(n+1)} \end{bmatrix} = \begin{bmatrix} \phi_1(y_1^{(n)}, y_2^{(n)}, \ldots, y_{d_L}^{(n)}) \\ \phi_2(y_1^{(n)}, y_2^{(n)}, \ldots, y_{d_L}^{(n)}) \\ \vdots \\ \phi_{d_L}(y_1^{(n)}, y_2^{(n)}, \ldots, y_{d_L}^{(n)}) \end{bmatrix}. \quad (6)$$

The experimentalist's dilemma is that for a system with d_L degrees of freedom, it seems to be necessary to measure d_L independent variables $y_1 \ldots y_{d_L}$, an almost impossible chore for a complex system. This problem found a solution in the embedding theorem of Takens [4]. For a series $x(n)$, n = 1,2,3… one can construct a global trajectory matrix

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_K \end{bmatrix} = \begin{bmatrix} x(1) & x(1+\Delta n) & \ldots & x(1+(d_E-1)\cdot\Delta n) \\ x(2) & x(2+\Delta n) & \ldots & x(2+(d_E-1)\cdot\Delta n) \\ \vdots & \vdots & \vdots & \vdots \\ x(K) & x(K+\Delta n) & \ldots & x(K+(d_E-1)\cdot\Delta n) \end{bmatrix}, \quad (7)$$

where d_E is the global embedding dimension and Δn is the characteristic sample lag. Every line X_k in this matrix assigns one point in the d_E – dimensional phase space and is one sample of the dynamic, as exemplified in Figure 3. Hence, the single-dimensional data have been transformed into a multi-dimensional phase space. Takens has shown that the resulting phase space is topological equivalent to the original state space of the physical process, while we can assume that all variables are generically connected in a nonlinear process [5]. From this phase space one will find out, if there exists a low-dimensional dynamic. More precisely, if one finds a subspace of the phase space where all system orbits converge, one will find the *attractor* of the system. Investigations of such an attractor are the key for understanding the underlying dynamic. The results are important invariants to find a model. But first of all the optimal embedding parameters d_E and Δn must be found. Therefor, the most common methods are explained below:

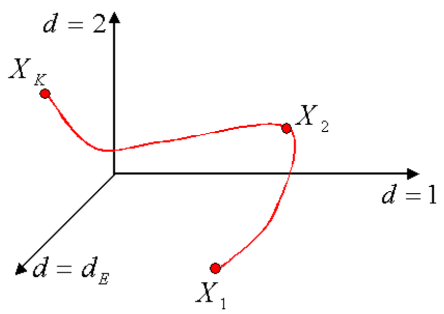

Figure 3: Phase-space reconstruction. The points X_k are specified in the global trajectory matrix.

a) The characteristic sample lag Δn

In the majority of cases one finds in literature the *average mutual information* to determine the characteristic sample lag (e.g. [5], [6]):

$$I(\Delta n) = \sum_{x(n),x(n+\Delta n)} P(x(n),x(n+\Delta n)) \cdot \log_2\left[\frac{P(x(n),x(n+\Delta n))}{P(x(n))\cdot P(x(n+\Delta n))}\right], \quad (8)$$

where $P(x(n),x(n+\Delta n))$ is the joint probability density for measurements $x(n)$ and $x(n+\Delta n)$. $P(x(n))$ and $P(x(n+\Delta n))$ are the individual probability densities for the measurements of $x(n)$ and of $x(n+\Delta n)$. In [7], Fraser suggests that one uses the first minimum of the average mutual information function because this function is a kind of nonlinear autocorrelation function to determine when the values of $x(n)$ and $x(n+\Delta n)$ are independent enough of each other to be useful as coordinates in a delay vector, but not so independent as to have no connection with each other at all [5].

Choosing the first minimum of the average mutual information seems to be similar to the choice of the first zero of the linear autocorrelation function, which is the optimum linear choice from the point of view of predictability in a least square sense. Nevertheless, as many investigations of many researchers have shown, the average mutual information is the better choice.

b) The global embedding dimension d_E

Modeling a process by the use of the theory of dynamics is not possible without the understanding of different kinds of dimensions. First of all, there is the dimension Takens has used for his embedding theorem. He has shown that a phase-space reconstruction by delay embedding will be topological equivalent to the original state space of the physical system, if the integer embedding dimension is $d_T > 2\cdot d_A$, where d_A is the dimension of the attractor. If one chooses this dimension d_T, it is sure that all orbits are fully unfolded. But often, such a high dimension is not needed to ensure this. We wish to determine the integer *global embedding dimension* $d_E \le d_T$, where we have the necessary number of coordinates to unfold observed orbits from self overlaps arising from projection on the attractor to a lower dimensional space.

A useful algorithm to determine this dimension is the *global false nearest neighbors* method which is described e.g. in [5] and [8]. Therefor, a low dimension d is chosen. For each point X_k in the associated phase space the nearest neighbor X_k^{NN} must be found, which has the minimal geometrical distance to this point. The label k of this nearest neighbor should bear little relation to the label k where X_k appears.

If X_k^{NN} is a true neighbor of X_k, then it came to the neighborhood through dynamical origins. If it is a false neighbor having arrived in its neighborhood by projection from a higher dimension, because the present

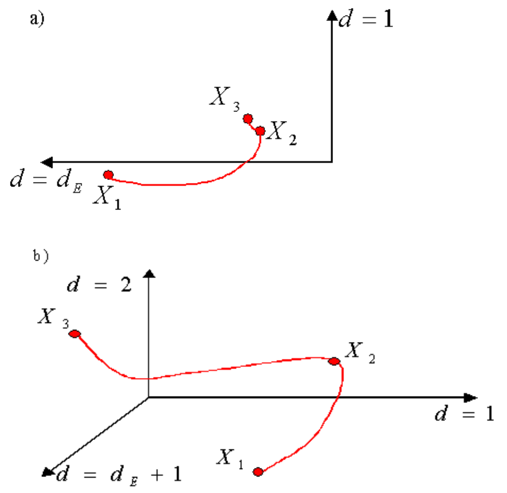

Figure 4: a) in the d_E-dimensional phase space X_2 and X_3 seem to be true neighbors; b) increasing the dimension to $d_E + 1$ shows that X_2 and X_3 are false neighbors.

dimension d_E does not unfold the attractor, then by going to the next dimension $d_E + 1$ this false neighbor may be moved out of the neighborhood of X_k, as exemplified in Figure 4. By looking at every data point X_k and asking at what dimension a minimum number of false neighbors can be found, one can find the global embedding dimension d_E.

3. Invariants of the Dynamics

a) The attractor dimension d_A

We have mentioned before that the attractor of the dynamical system is a subspace of the phase space, wherein all system orbits converge. More precisely, it is a set of points which are still reached by orbits after a (theoretically) infinite number of samples along label k. The attractor can be called the most important structure in nonlinear signal processing, because all dynamical invariants are derived from its base. The dimension d_A of this attractor does not need to be an integer number. Especially chaotic systems often have a *fractal* attractor-geometry [17]. The reason for this is that orbits in chaotic systems are averaged diverging exponentially in at least one variable. But at the same time the attractor has to be bounded to a subspace of the phase space. The only possibility to fulfill both conditions at the same time, is that the diverging orbits are folded back to the space of the attractor. This behavior is often shaping a fractal geometry. To evaluate the fractal dimension d_A Grassberger [9] developed a method using the correlation integral

$$C(r,d) := \lim_{K\to\infty} \frac{1}{K^2}\cdot \sum_{\substack{i,j=1 \\ i\neq j}}^{K} \Theta\left(r - \left\|X_i - X_j\right\|\right), \quad (9)$$

where $\Theta(.)$ is the Heaviside function and r is a threshold for the distance between the points X_i and X_j. Drawing the result of the logarithm of the correlation integral against the logarithm of r, one gets the correlation dimension d_2 by evaluating the gradient of the plot for small r:

$$d_2 := \lim_{r\to 0} \frac{\partial[\log(C(r,d))]}{\partial[\log(r)]} \quad (10)$$

If this value converges for high d, a good approximation for the fractal dimension of the attractor $d_A \approx d_2$ has been found.

b) The local embedding dimension d_L

In terms of reconstructing the geometry of the attractor in a topological equivalent phase space, in theory it is impossible to choose an embedding dimension which is too high. Hence, Takens' embedding dimension is the lowest dimension reconstructing the geometry of the attractor in any case, even though it is often not the optimum. Due to limited processing power and workspace demand of our computers, we are always keen to find a minimal embedding dimension, which enables us to reconstruct the attractor's geometry. This is why we have introduced the method of global false nearest neighbors to find the global embedding dimension d_E, as explained above.

To find out, how many independent equations are representing the system's dynamic, it is necessary to see whether locally one requires fewer dimension than d_E. This means that the evolution of the orbits should be considered as they move on the attractor. Often there is a necessary dimension $d_L \le d_E$. It is called the local embedding dimension or the "dynamical" dimension and is the number of *active degrees of freedom*. The examination of this dimension must be done very carefully, because it identifies the correct number of independent equations representing the system's dynamic. For this purpose, it can be applied the *local false nearest neighbors* algorithm [10]. This approach works in a global dimension which is large enough to assure that all neighbors are true, namely, some working dimension $d_W \ge d_T$. In this space, one moves to a point X_k on the attractor and ask what subspace of dimension $d_L \le d_T$ allows one to make accurate local neighborhood to neighborhood maps of the data on the attractor. For this, the N_B nearest neighbors of X_k are chosen and the eigenvalues and eigendirections of the associated covariance matrix are calculated. By choosing the d_L eigendirections with the largest eigenvalues the projection of the data vectors X_k onto this local basis can be formed. Now, one only need to find the one nearest neighbor of the projection of the point X_k in that subspace. After that we return to the d_W–dimensional space and calculate the distance between the nearest neighbor found in the d_L–dimensional space and X_k. By considering the evolution of this distance along label k, the right dynamical dimension d_L can be found out, namely the dimension where this distance evolution becomes independent of d_L and N_B.

c) The Lyapunov spectrum

One of the most important dynamical invariants is the *Lyapunov spectrum*. It consists of a set of d_L *Lyapunov exponents* $\lambda = [\lambda_1 > \lambda_2 > ... > \lambda_{d_L}]$, one exponent per active dynamical degree of freedom. The Lyapunov exponents are values, which compare the distance evolution of orbits having neighboring starting-points in the d_L-dimensional phase space along the label k with exponential growing. If the distance is growing exponentially, that means two orbits which are nearest neighbors in their starting points are diverging very fast, the Lyapunov exponent will be positive. Does an observed process has at least one positive Lyapunov exponent, the underlying system is chaotic. If any of the Lyapunov exponents is zero, there is a high likelihood that we have differential equations describing the dynamics. The sum of the Lyapunov exponents must be $\lambda_1 + \lambda_2 + ... + \lambda_{d_L} < 0$, because our considered system should have a dissipative nature [11],[12].

To determine the Lyapunov exponents of a data series, the change of a little part of the attractor's volume has to be considered along the label k. This little volume part is usually assumed to be a hyper-sphere with radius ε. If ε is small enough, this change can be approximated by the linearized dynamics J_k, which can be derived by linearizing the global trajectory matrix by a least mean square approach. Due to the influence of this linearized dynamics the hyper-sphere is deformed into a hyper-ellipsoid with d_L axes $\varepsilon_i = \sigma_i \cdot \varepsilon$, as shown in Figure 5, where σ_i are the singular values of J_k.. The Lyapunov exponents are defined as

$$\lambda_i := \lim_{k\cdot\Delta\to\infty} \frac{1}{k\cdot\Delta} \ln(\sigma_i(k)), \quad (11)$$

where Δ is the sample period of the observed data. Because the numerical determination of the singular values of small matrices is not trivial, the method is based on a QR-decomposition. A detailed explanation of a numerical method to calculate the Lyapunov exponents of data series is given by Eckmann [13].

4. Analyzing the Mobile Radio Channel

During this first analysis campaign our investigations have been focused on simulated data series generated by a ray-tracing simulator. This simulator has been developed for analyzing spectral extrapolation algorithms and channel sounder concepts, and has been verified by measurements. Reasons for using simulated

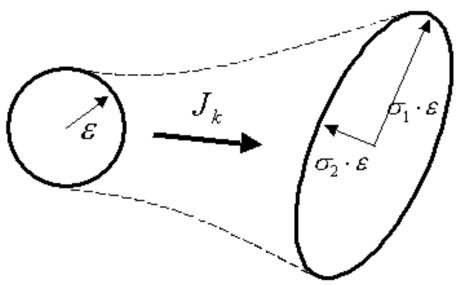

Figure 5: Deformation of the hyper-sphere into a hyper-ellipsoid due to the linearized dynamics J_k.

data, at first, are that the source generating these data is comprehensible at any time and is not contaminated by noise. Thus, a preliminary analysis of simulated data enables us to distinguish between effects forced by an underlying dynamic, only, and a process which consists of a dynamic and additive noise, like measurement data, which will be analyzed in a second step.

The results in this paper are based on a scenario adapted to a long street within an urban environment according to Figure 6, where additionally the most important parameters are given for simulation.

Since geometrical dimensions are very large with respect to the wavelength, diffraction effects have been neglected, whereas reflections have been considered up to an order of 100. In a first step, our analysis has been focused on the impulse response at a fixed point in observation time t_0. Therefore, all results in this paper are based on multi-path propagation with fixed transmitter and receiver locations. If it is possible to verify some dynamical characteristics of this $h_{Rx}(\tau,t_0)$, it must be possible to the time-variant channel, too, because this multi-path propagation is one of the most fundamental process of the mobile radio channel.

The dynamical analysis of experimental data series requires stationary data sets [14]. However, the considered multi-path propagation in terms of impulse responses of the time-invariant radio channel is not stationary because the amplitude is decreasing in a kind of exponentially manner within the delay-time domain. To overcome this problem a Fourier transform has been carried out. Fortunately, the analyzed data was stationary in the frequency domain, as it has been shown by applying the *recurrence-plot analysis* by Eckmann [15]. Figure 7 depicts the results of two recurrence-plot analyses. The upper one is the result of the received signal's real part of the *z*-component in the delay-time domain $\mathrm{Re}\{h_{Rx,z}(\tau,t_0)\}$, the lower one shows the result of the same signal in the frequency domain $\mathrm{Re}\{H_{Rx,z}(f,t_0)\}$, where $H_{Rx,z}(f,t_0)$ is the Fourier transform of $h_{Rx,z}(\tau,t_0)$. While the recurrence

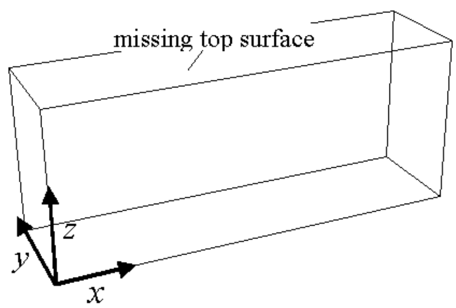

Figure 6: Scenario for considered ray-tracing simulation.

size:	1000m / 15m / 20m
material:	concrete ($\varepsilon_r = 6.5$; $\tan\delta = 0.0659$)
air attenuation :	0 dB
transmitter position:	320m / 1m / 19m
receiver-position:	820m / 7m / 2m
carrier frequency:	1 GHz
antennas:	infinitesimal dipole
polarization	*z* - direction

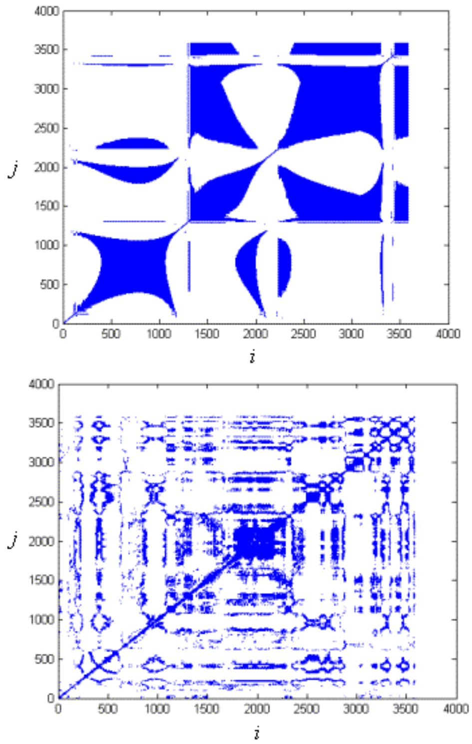

Figure. 7: Upper fig.: recurrence plot of the receiver signal's real part of the *z*-component in the delay-time domain $\mathrm{Re}\{h_{Rx,z}(\tau,t_0)\}$; Lower fig.: recurrence plot of the same signal in the frequency domain $\mathrm{Re}\{H_{Rx,z}(\tau,t_0)\}$; *i* and *j* are the indices of the phase space's points.

plot of the signal in the delay-time domain shows obvious changes in the density of the pattern, the recurrence plot of the signal in frequency domain appears fairly uniform filled out within the complete surface. In literature, this characteristic is considered as an evidence for stationary data [6], [13].

Analyzing data series in frequency domain might be an unaccustomed perception for someone who is adept to the theory of dynamics, but for a communication engineer it is irrelevant to model the transfer function in frequency-domain or the impulse response in time-domain. Hence, we use the dynamical analysis in frequency domain as a powerful mathematical tool.

5. Results

We have analyzed in-phase- and quadrature-components of three different spatial components (*x*, *y*, *z*) of the received signal. All these six data series of the ray-tracing simulator have been reduced to a bandwidth of 1 GHz and to 3500 samples after a discrete Fourier transform. Because the bandwidth is in the same range as the carrier frequency of 1 GHz, we can definitely assume a broadband channel. The results have shown that the data seems to be caused by a low-dimensional process, because we have got correlation dimensions between $d_2 = 3 \ldots 4$ and dynamical dimensions of $d_L = 4$ for all considered data series, as we can see in Table 1. These results are very different from random processes, because these processes theoretically have infinite degrees of freedom. Hence, it is possible to

Frequency series	Correlation dimension d_2	Lyapunov-spectrum λ_i
$\mathrm{Re}\{H_{Rx,x}(f,t_0)\}$	3.1681	+ ≈ 0 − −
$\mathrm{Im}\{H_{Rx,x}(f,t_0)\}$	3.4239	+ ≈ 0 − −
$\mathrm{Re}\{H_{Rx,y}(f,t_0)\}$	3.5408	+ ≈ 0 − −
$\mathrm{Im}\{H_{Rx,y}(f,t_0)\}$	3.6003	+ ≈ 0 − −
$\mathrm{Re}\{H_{Rx,z}(f,t_0)\}$	3.8107	+ ≈ 0 − −
$\mathrm{Im}\{H_{Rx,z}(f,t_0)\}$	3.9301	+ ≈ 0 − −

Table 1: Results of the dynamical analysis. It has been obtained $\Delta n = 1$ and $d_L = 4$ for every data series. +: positive Lyapunov exponent; -: negative Lyapunov exponent; ≈ 0: Lyapunov exponent around zero.

distinguish between our reviewed process and a stochastic one.

Furthermore, the calculation of the Lyapunov exponents has yielded for each of the data series one positive, two negative and one Lyapunov exponent in the range of zero. The sum of all Lyapunov exponents per data set has always been smaller than zero. These results imply, that there is an underlying chaotic dissipative system providing a high likelihood that there are differential equations, which describe the dynamics of the mobile radio channel.

6. Conclusion and Outlook

In this paper, we have shown that alternative methods of modeling the mobile radio channel are applicable. This is motivated, because the traditionally used linear stochastic models can be harmed at high bandwidths, by existing correlations between different propagation paths or by the use of high-gain antennas for spatial filtering purposes.

Our approach is based on the idea that there exists an underlying low-dimensional process which causes the physical behavior of the mobile radio channel. By applying analysis methods of the theory of dynamics, we want to find a low-dimensional model of the mobile radio channel.

The main focus of this paper has been the introduction of typical dynamical analyzing methods, which are different from linear stochastic approaches. The advantage of dynamical models is that they are going beyond statistics. While statistics has an infinite number of degrees of freedom, a dynamical model needs only a low dimension of the phase space. Hence, the primarily goal in terms of applying nonlinear analysis is searching for an underlying *low-dimensional* process which causes the output of the system. Specifically, for a given set of data, we will construct a dynamical model with a set of deterministic differential or difference equations, which replaces the process generating these data.

The results have shown that **there is** an underlying low-dimensional chaotic process, which can be used to reconstruct the output of the ray-tracing simulation for a scenario adapted to a mobile radio channel within an urban environment. These results encourage us to go on with our investigations of modeling the mobile radio channel by dynamical models. Analyzing comprehensive measurement data from different measurement campaigns and creating a construction algorithm which generates a set of differential or difference equations based on the results in this paper, are the next steps.

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