# Space-Time QPSK Trellis Codes With Unequal Error-Protection Properties

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***Abstract*: The multiple input bits of a space-time trellis code are unequally protected from errors. In this paper, this property is explored by searching for ST trellis codes that provide very strong error protection properties to one of the input bits. This is done using an exhaustive search approach that is applied on 4, 8 and 16-State QPSK codes. The QPSK codes found using this method are simulated and their performance is compared to some of the codes in the literature with improvement reaching up to 1 dB.**

## 1. Introduction

*Space-time* (ST) trellis codes (first introduced in their present form by Tarokh *et al* in [7]) rely on the idea of combining multiple transmit antennas with trellis-coded modulation. This resulted in a coding technique that provides both high transmission rate and strong error control capabilities for fading channels. Performance evaluation of ST trellis codes over rapid fading channels depends on their distance properties [7].

For certain applications, it is desirable that the most important part of information has better error protection. In such applications, a code with with very strong error protection on the most significant input bit (MSB) can outperform a code with strong overall error protection, but with equal error protection on all input bits. Since ST trellis codes have more than one input bit into the encoder, it is possible to concentrate the strength of error protection on one of the input bits. In order to satisfy this goal, a search approach is proposed to find codes that give more protection on one input bit.

The following section describes the system model to be used in this work. Section 3. develops the code generation and code search criteria. The new codes and their performance compared to codes from the literature are shown in Section 4.. Finally, some conclusions are drawn in the last section.

## 2. System Model

In a typical ST trellis encoder, $k$ input bits are encoded to $n \times \tilde{n}$ coded bits. These bits are divided into $n$ sets, each consisting of $\tilde{n}$ bits. Every $\tilde{n}$ bits set is then mapped onto a point from a $2^{\tilde{n}}$-ary signal set. Finally, the resulting $n$ symbols are modulated and transmitted at the same time instance and at the same frequency. The decoder consists of $m$ receive antennas passing the received signals to $m$ demodulators then to a maximal-ratio combiner and a Viterbi decoder.

The received signal $r_t^j$ at the $j^{th}$ antenna at time $t$ is a noisy superposition of all transmitted symbols over all transmit antennas and is given by:

$$r_t^j = \sum_{i=1}^{n} \alpha_{i,j}(t) c_t^i \sqrt{E_s} + \eta_t^j \qquad (1)$$

where $\eta_t^i$ is an AWGN modeled as independent samples of a zero-mean complex Gaussian random process with variance $N_0/2$ per dimension. The coefficient $\alpha_{i,j}(t)$ is the path gain from the $i^{th}$ transmit antenna to the $j^{th}$ receive antenna and $c_t^i$ is the transmitted symbol from the $i^{th}$ transmit antenna at time $t$. For the performance evaluation in this section a rapid fading channel with independent fade coefficients (i.e. fade coefficients change independently, with time, from one fade coefficient to another) is going to be considered.

At the receiver side, the Viterbi decoder computes a branch metric defined by the following:

$$\sum_{j=1}^{m} |r_t^j - \sum_{i=1}^{n} \alpha_{i,j}(t)\, c_t^i|^2 \qquad (2)$$

For a general trellis code, bit error probability is given by [5]:

$$P_b \leq \frac{1}{k} \cdot \frac{\partial T(D,I)}{\partial I}\bigg|_{I=1} \qquad (3)$$

where $T(D,I)$ is the *transfer function* as a function of the distance enumerator $D$ and the input bits enumerator $I$. The transfer function of an *error-state diagram* with $N$ states is a function that characterizes the code by enumerating all possible error sequences. The error-state diagram is derived from the code state diagram by labeling its branches with error weight profile. The transfer function of quasi-regular codes can be represented as [5]:

$$T(D,I) = \sum_{l=1}^{\infty} \sum_{\mathbf{E}_l \neq 0} I^i \prod_{t \in \hat{\nu}} D \qquad (4)$$

The transfer function in the form $T(D,I)$ does not show other characteristics such as the magnitude of protection on individual input bits (depending on their position). Therefore, a modified version of it is used to reflect the weight of individual input bits [3]:

$$T(D,I_1,I_2,\ldots,I_k) = \sum_{l=1}^{\infty} \sum_{\mathbf{E}_l \neq 0} I_1^{i_1} I_2^{i_2} \cdots I_k^{i_k} \prod_{t \in \hat{\nu}} D \qquad (5)$$

where $I_1, I_2, \ldots, I_k$ denote the individual input bits enumerators.

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To find the BEP of individual input bits, consider the modified transfer function of Equation (5) in a manner similar to the work in [6]. The new upper-bound for individual input bits becomes [3]:

$$P_b^{(i)} \leq \left.\frac{\partial T(D, I_1, I_2, \ldots, I_k)}{\partial I_i}\right|_{I_i=1} \qquad (6)$$

where $P_b^{(i)}$ is the average BEP of the input bit $i$.

## 3. Exhaustive Search Approach

The first step in the exhaustive search approach is code generation. After generating all possible codes, the codes are searched to find the best ones satisfying the proposed search criteria. Finally, the chosen codes are simulated to check their performance and see if they satisfy the expectations.

### 3.1. Code Generation

In order to generate all possible ST trellis codes of a certain number of states and modulation scheme, one must reside to a representation other than the trellis representation. The reason is that the trellis representation is very general. For example, it includes non-linear codes which are beyond the scope of this work. One possible alternative representation is described in the following.

Let $\mathbf{I}$ be a binary vector of input bits defined as

$$\mathbf{I} = \mathbf{i}_0\mathbf{i}_1 \cdots \mathbf{i}_t$$

where

$$\mathbf{i}_t = i_t^{k+j}\ i_t^{k+j-1} \cdots i_t^0$$

where $k$ is the number of input bits to the encoder in parallel and $j$ is the number of memory elements. Let $\mathbf{G}$ be a generator matrix of size $(k+j) \times (n\tilde{n})$ where $n\tilde{n}$ is the number of encoded bits. Each entry in $\mathbf{G}$ is either 0 or 1 depending on the presence or absence of a connection between the output bits and the input bits. If a column has more than one entry of 1, it the connections are combined using modulo-2 addition. The reason of using a modulo-2 adder is to ensure the linearity of the encoder which is a quasi-regularity requirement (*see* [2] for the conditions of quasi-regularity). The corresponding binary encoded-bits vector at time $t$ $\mathbf{a}_t = a^{n\tilde{n}}\ a_t^{n\tilde{n}-1} \cdots a_t^0$ is given by:

$$\mathbf{a}_t = \mathbf{i}_t \cdot \mathbf{G} \qquad (7)$$

Using this representation, all possible *linear* codes (for a given $n\tilde{n}$, $k$ and $j$) can be generated by having all possible combinations of $\mathbf{G}$. The trellis representation of a certain code can be produced by mapping the output binary codewords $\mathbf{a}$ of all possible different input sequences using a one-to-one mapping function $\mu$.

### 3.2. Code Search Criteria

After generating all possible codes with certain $n\tilde{n}$, $k$ and $j$, it is necessary to define the criteria for choosing the best codes. As pointed out earlier, the goal of this work is to find codes that provide strong error protection to the MSB. To accomplish this task for quasi-regular codes, it is possible to depend on the new upper-bound of individual input bits of Equation (6). The reason why the conventional distance and product criteria are not used in their original form is that they do not reflect the weight of protection on different input bits. Furthermore, for codes with equal minimum symbol-differences and minimum product distance properties, it is necessary to go to higher product distances.

In order to find codes of strong error protection on one input bit, it is necessary to apply the following rules in the search:

1. Codes that do not span the whole codewords spectrum are not considered.
2. Inter-codeword squared Euclidean distances (SED) in each state are maximized.
3. $P_b^{(i)}$ of the chosen MSB is maximized.

Item 1 is a condition to reject codes that do not span the whole codeword spectrum (16 codewords for QPSK). Such codes are obviously weak in performance. Item 2 strengthens the performance of the code in general [8]. Finally, finding codes with strong error protection on the MSB relyes on Item 3.

## 4. The New Codes and Their Performance

In this section, the proposed search criteria is applied to 4, 8 and 16-state QPSK (The method has been also applied to 8-state 8PSK in in [1]). Then the new codes are simulated over a rapid channel and compared to some of the codes in [4] (referred to as F-V-Y) and [8] (referred to as Z-S). The exhaustive search results are shown in Figures 1, 2 and 3, respectively.

Table 1 shows minimum product distance comparison of the MSB and LSB for the new codes and various codes from the literature. It is clear that there is a significant increase in the minimum product distance for the MSB In the new codes compared to the ones in the literature. This is proven in the simulation plots shown in Figures 4, 5 and 6 which show an improvement for the MSB in the new codes compared to other codes in the literature. As for the LSB, there is a decline in the minimum product distance properties, which shows in the simulation plots. This is expected since, as was mentioned earlier, the MSB was the center of focus, while the LSB was ignored totally. The improvement is relatively low at low SNR values, but the gain increases at high SNR. The largest gain is for the QPSK 8-state code, which gives a gain of more than 1.5 dB over the nearest code at a bit error rate BER $= 10^{-5}$. In the cases of 4 and 16-state QPSK, the gain is around 1.0 and 0.75 dB at BER $= 10^{-5}$, respectively.

## 5. Summary and Conclusions

In this paper we proposed a new search method. The method was demonstrated by finding several 4, 8 and 16-state QPSK codes that provide strong error protection to the MSB. All codes were simulated on a rapid fading channel and the results were presented and compared to existing codes. All codes gave strong protection to the MSB and an improvement that reached more than 1 dB over other codes in the literature. Moreover, this method is applicable to higher constellations.

Table 1: Minimum product distance comparisons of the MSB and LSB for various codes

| | | F-V-Y | Z-S | New |
|---|---|---|---|---|
| M | 4s QPSK | 32.0 | 32.0 | 36.0 |
| S | 8s QPSK | 32.0 | 96.0 | 144.0 |
| B | 16s QPSK | 96.0 | N/A | 216.0 |
| L | 4s QPSK | 24.0 | 24.0 | 8.0 |
| S | 8s QPSK | 48.0 | 24.0 | 8.0 |
| B | 16s QPSK | 64.0 | N/A | 32.0 |

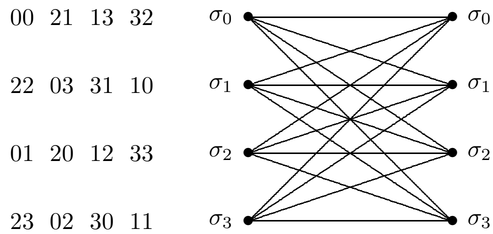


Figure 1: 4-State QPSK code

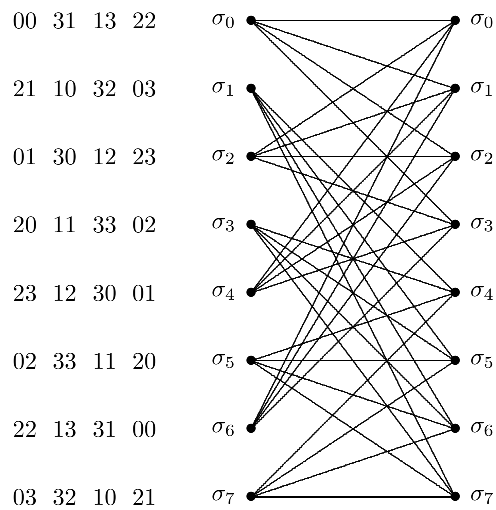


Figure 2: 8-State QPSK code

## REFERENCES


[1] Hesham M. Al-Salman. Application of space-time trellis codes for image transmission over wireless channels. Master's thesis, KFUPM, 2003.

[2] Hesham M. Al-Salman and Saud A. Al-Semari. Distance properties of space-time trellis codes. *International Symposium of Wireless Systems and Networks*, Mar 2003.

[3] Hesham M. Al-Salman and Saud A. Al-Semari. Distance spectrum of space-time codes. *Arabian Journal for Science and Engineering - Special Issue on Wireless Systems and Networks*, Apr 2003. submitted.

[4] Welly Firmanto, Branka S. Vucetic, and Jinhong Yuan. Space-time TCM with improved performance on fast fading channels. *IEEE Communication Letters*, 5(4):154–156, Apr 2001.

[5] S. H. Jamali and T. Le-Ngoc. *Coded-Modulation Techniques for Fading Channels*. Massachsetts: Kluwer Academic Publisher, 1994.

[6] D. G. Mills and D. J. Costello. Using a modified transfer function to calculate unequal error protection capabilities of convolutional codes. *IEEE Proceedings on Information Theory*, page 144, 1993.

[7] Vahid Tarokh, Nambi Seshadri, and A. R. Calderbank. Space-time codes for high data rate wireless communication: Performance criterion and code construction. *IEEE Transactions on Information Theory*, 44(2):744–765, Mar 1998.

[8] Salam A. Zummo and Saud A. Al-Semari. Space-time coded QPSK for rapid fading channels. *PIMRC*, 1(1):504–508, Sep 2000.

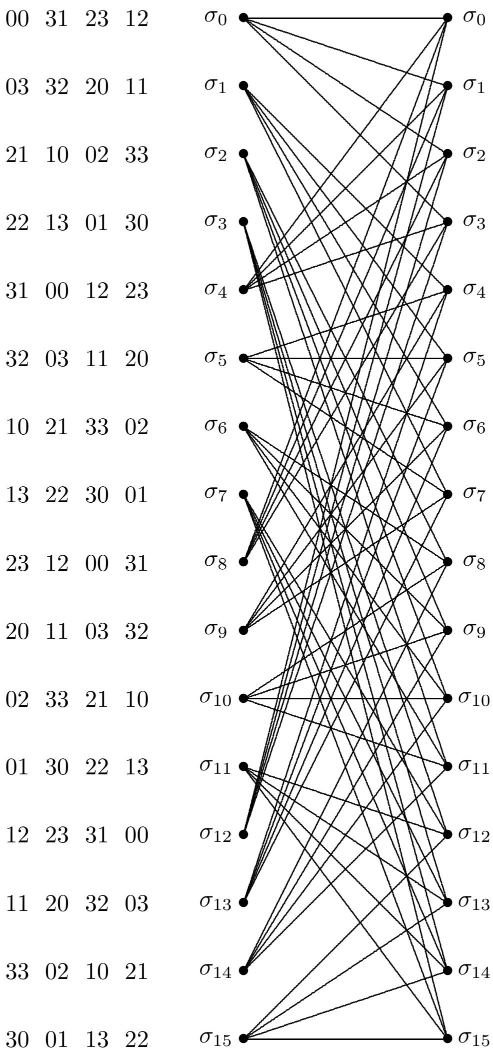


Figure 3: 16-State QPSK code

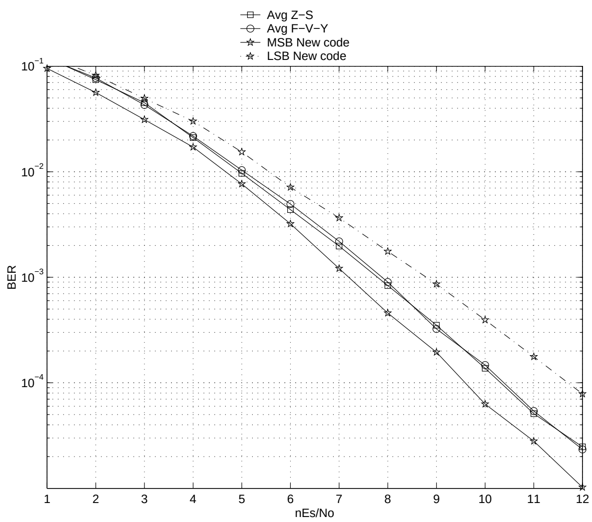


Figure 4: Simulation results for the new codes and codes from the literature (QPSK 4-state, 2 receive antennas)

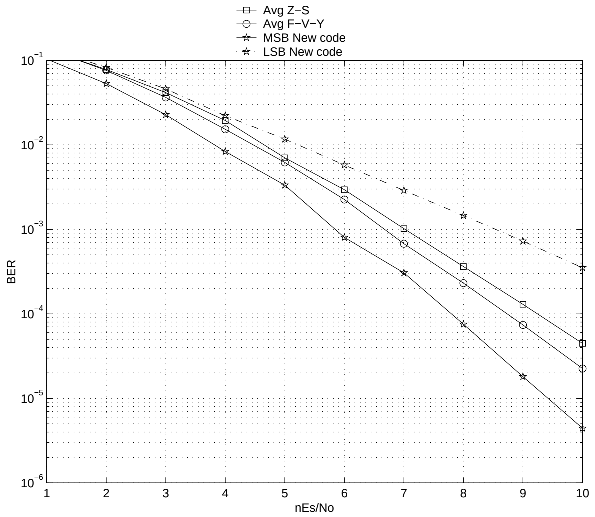


Figure 5: Simulation results for the new codes and codes from the literature (QPSK 8-state, 2 receive antennas)

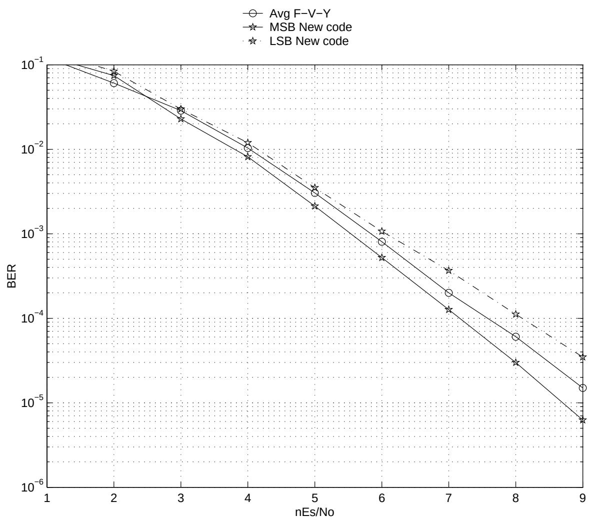


Figure 6: Simulation results for the new codes and codes from the literature (QPSK 16-state, 2 receive antennas)